Sub:

Date: 1/1/2024

Graphs

Completing the table

Plotting the points to form a graph

Solving the equations by using the graphs.

finding the gradient of a tangent to a curves.

ex: $f(x) = x^3 - 2x^2 + 3$

*like this or use table mode in calc

$f(-3) = (-3)^3 - 2(-3)^2 + 3$

$f(-2) = (-2)^3 - 2(-2)^2 + 3$

x	-3	-2	-1	0	1	2
y	-42	-13	0	3	2	3

if has gaps:

x	-3	-2	-0.5	1.25	3
y					

we can substitute values into formula.

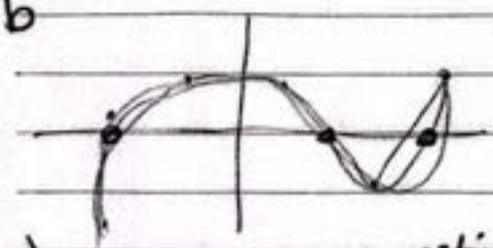
OR

if missing values are integers, we can solve normally using calculator.

a

x	-3	-2	-0.5	0	1	2	2.5	3
y	4		✓			✓	✓	

b



case1

c

1) from the graph, estimate solve $x^3 - 2x^2 + 4 = 0$

where curve meets x-axis

$x =$ ____, $x =$ ____, $x =$ ____

2) solve $f(x) = 0$

Solve $y = 0$

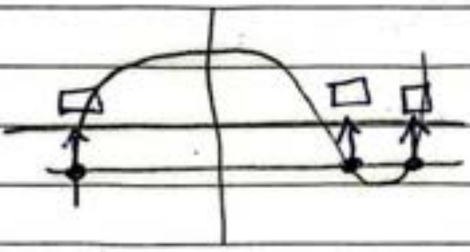
case2 *we are looking for points of intersection

ii) from the graph estimate solve $x^3-2x^2+4=\overset{y}{3}$

→ x values (can be less or more)

if there is a diff equation, we subtract formulas & do the same.

solve $x^3-2x^2+7=0$



old | x^3-2x^2+4

− new | x^3-2x^2+7

$y=-3$

case3

iii) by drawing suitable straight line, solve $x^3-2x^2+x-5=0$

old | $\cancel{x^3-2x^2}$ 0+4

new | $\cancel{x^3-2x^2}+x-5$

$\boxed{y=-x+9}$

*if same formula in both parts, then we solve directly, if different then old-new

case4

iv) by drawing suitable straight line, solve $2x^3-4x^2+10=$

*we must make sure that changes are 8 $y=0$

& *we must make sure that if there is a power over one, then they must have same coefficient.

$y=0$
$y=\square$
$y=mx+c$

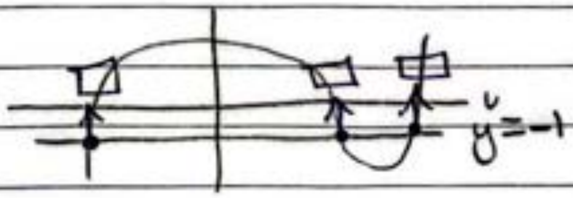
$\frac{2x^3}{2}-\frac{4x^2}{2}+\frac{10}{2}=0$

$x^3-2x^2+5=0$] because main is x^3-2x^2+4

now, old | $\cancel{x^3}-\cancel{2x^2}+4$

new | $\cancel{x^3}-\cancel{2x^2}+5$

$y=-1$



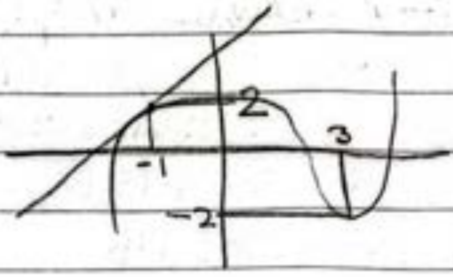
x= ___ , x= ___ , x= ___

finding gradient of a tangent to a curve

*the line that touches the edge of the curve

find gradient of tangent at $x=3$.

$\frac{y_2-y_1}{x_2-x_1}$ (3,−2)
(0,0)



estimate gradient at $x=-1$
tangent

(−1,2) (–,–)

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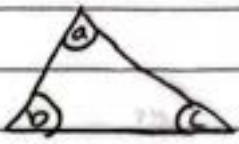
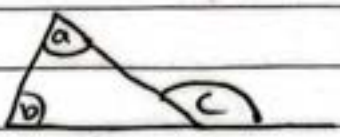
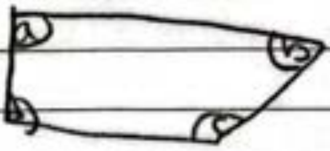
triangle $= a+b+c = 180^\circ$
angles in triangle add up to 180



$a+b=c$] exterior angle



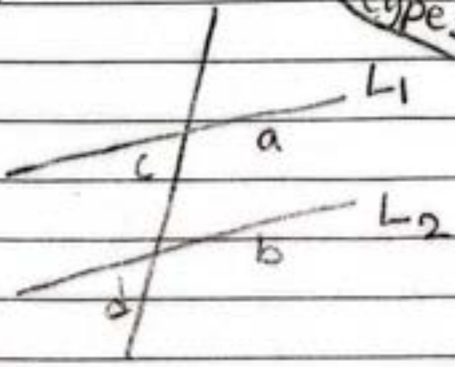
shapes with 4 sides
angles in a quadrilateral
$a+b+c+d = 360^\circ$



Parallel Lines :

E Z F

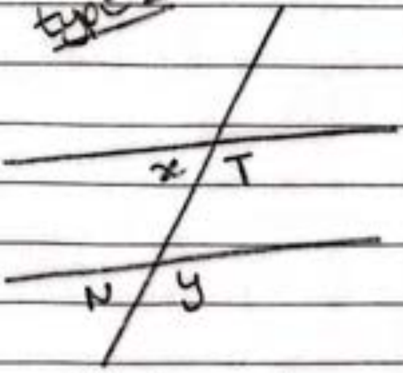
F
type 1

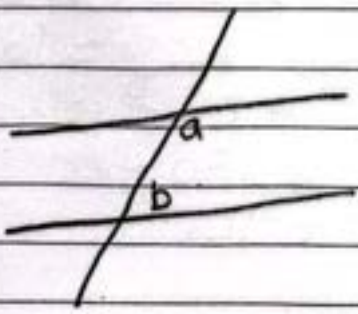


corresponding
$a=b$
$c=d$

alternate
$x=y$
$n=t$

Z
type 2





interior angle
$a+b = 180$

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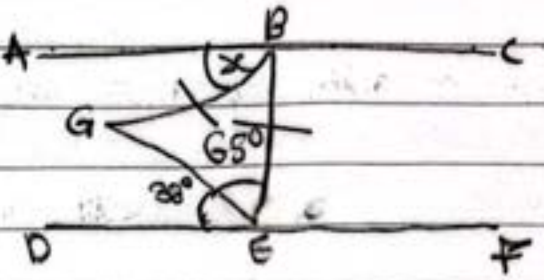
Q-ABC and DEF are parallel
BG = BE
$\angle DEG = 38^\circ$
$\angle GEB = 65^\circ$

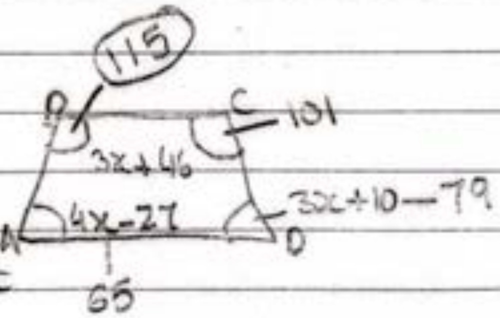


$\angle BGE = \angle BEG = 65$
$\angle GBE = 180 - (65+65) = 50$

$38 + 65 + 50 + x = 180$
$153 + x = 180$
$180 - 153 = x$
$\boxed{27^\circ = x}$

Q-BCD and AFE are straight
Show that BCD is parallel to AFE

Q-ABCD trapezium
BC is parallel to AD
find size of largest angle. $\boxed{B}$



$(3x+46) + (4x-27) = 180$
$7x + 19 = 180$
$7x = 180 - 19$
$\frac{7x}{7} = \frac{161}{7}$
$x = 23$

$\boxed{3(23) + 46 = 115}$
$4(23) - 27 = 65$

$3x + 10 + c = 180$
$3(23) + 10 + c = 180$
$69 + 10 + c = 180$
$79 + c = 180$
$c = 180 - 79$
$c = 101^\circ$

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$\overbrace{\text{Poly}}^{\text{many}}\underbrace{\text{gons}}_{\text{corner}}$:

An 'n sided polygon can be split into $(n-2)$ triangles

The sum of interior angles in an n-sided polygon $= (n-2) \times 180°$

regular polygon	
all sides equal	
all angles equal	
sum of exterior angles is 360°	

exterior angle $= \frac{360°}{n}$

interior angle $= 180° - \frac{360°}{n}$

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Date: 5/1/2024

many

~~[illegible]~~

Geometry

Angles in a Polygon. → greek word

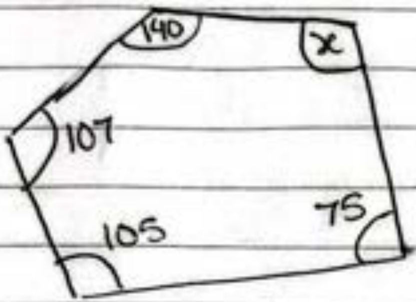
corners

triangle = sum of interior angles = 180°

~~Quadrilateral = sum of~~

sum of interior angles in an n-sided polygon = (n−2)×180

how many angles



find x

$\text{sum of angles} = (5-2)\times 180 = 540°$

$x+75+105+107+140 = 540$

$x + 427 = 540$

$x = 540 - 427$

$\boxed{x = 113}$

regular polygon	irregular polygon
all sides same length	
all angles equal	
sum of exterior angles is 360°	
interior + exterior = 180°	
exterior = $\frac{360}{n}$	
interior = 160 − $\underline{360}$ →	$\underline{(n-2)180}$

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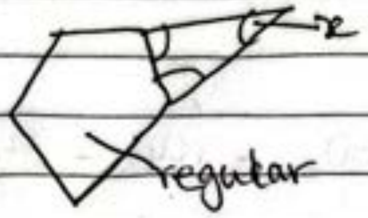
Q- ABCDE is a regular polygon
find x



long ways

$\frac{(n-2)\times180}{n}$

$\frac{(5-2)\times180}{5}$ = 108 → interior

Short way:

$\frac{360}{n}$

180 − 108 = 72 → exterior

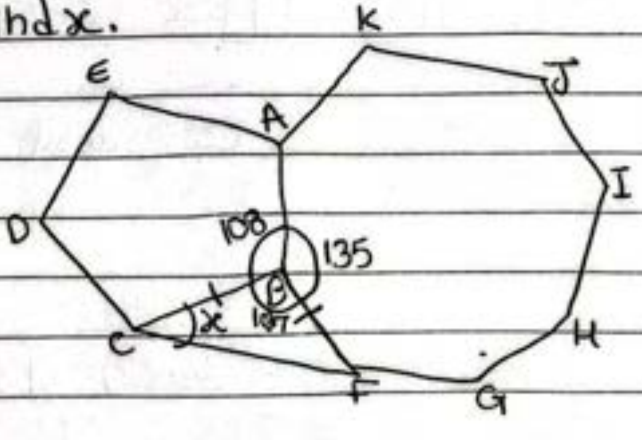
$\frac{360}{5}$ = 72 → exterior

72×2 = 144

180 − 144 = $\boxed{36}$ x

180 − (72×2) = $\boxed{36}$ x

Q- ABCDE is a regular polygon, ABFGHIJK a regular ocbagon, an isocsles triangle BCF.
findx.



$\frac{360}{5}$ = 72

$\frac{360}{8}$ = 45 → exterior

180 − 72 = 108

180−45 = 135 → interior

360 − (135+108) = 117

180 − 117 = 63

63/2 = $\boxed{31.5}$ x

isocsles

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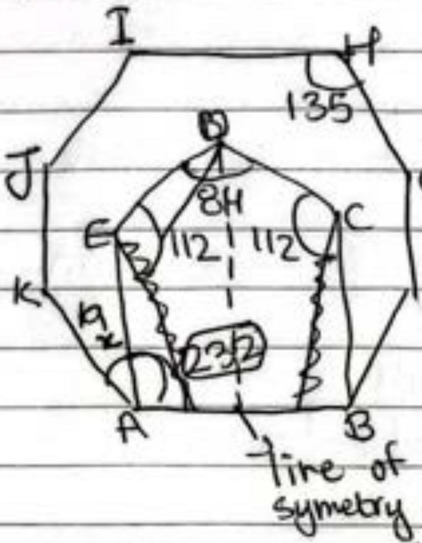
Q-Pentagon ABCDE is inside regular ~~pentagon~~ octagon ABFGHIJK.
The pentagon has one line of symetry.
find x.



octagon:

$$180 - \frac{360}{n} = 180 - \frac{360}{8}$$

$= 135 \rightarrow$ interior

pentagon:

$$\frac{(n-2) \times 180}{2} = \frac{(5-2) \times 180}{2}$$

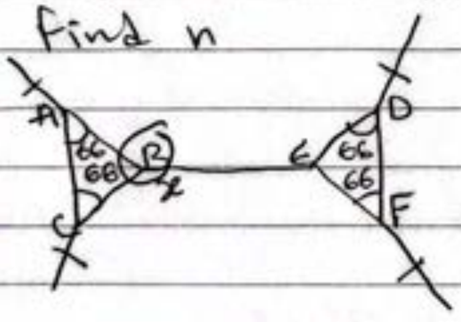
$= 540 - [112 + 112 + 84]$

$A + B = 232$

$$\frac{232}{2} = \boxed{116}$$

$135 - 116 = \boxed{19}\ x$

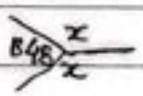
Q-2 $\overbrace{\text{congruent}}^{\text{same thing}}$ triangles & part of 2 congruent regular polygons, x & y.
both havex n sides
find n



$\angle ABC = 180 - (66 \times 2) = \boxed{48^\circ}$ B

$360 - 48 = 312$

$312/2 = \boxed{156}\ x$



interior + exterior = 180°

exterior = 180 − 156

exterior = $\boxed{24}$

$$n = \frac{360}{\text{ext.}}$$

$$n = \frac{360}{24} = \boxed{15}$$

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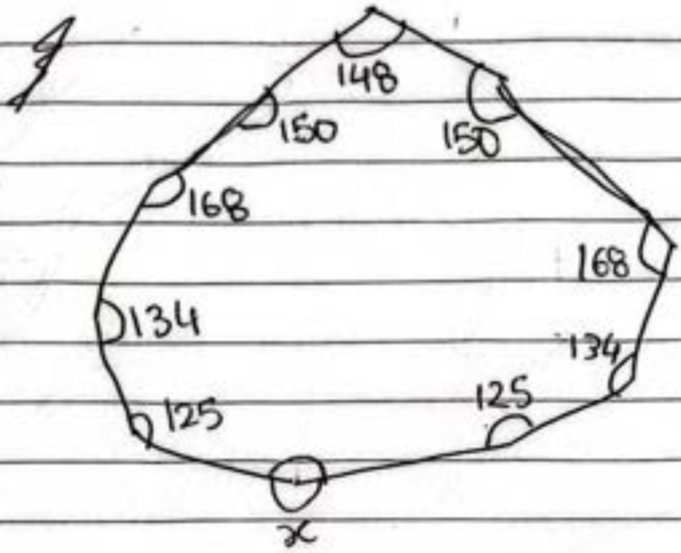
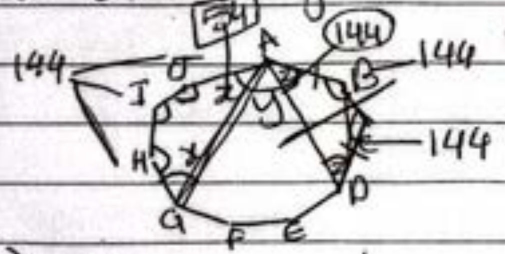
Q- here is a 10-sided polygon.
find x



Q- regular 10-sided polygon ABCDEFGHIJ
show that $x=y$



$$\frac{(n-2)180}{n} = \frac{(10-2)\times 180}{10}$$

$$= \boxed{144} \text{ interior}$$

$$\frac{(n-2)180}{}$$

$(5-2)180 = 540$

$2x + (144 \times 3) = 540$

$2x = 540 - 432$

$\boxed{x = 54}$

$2z + 144 + 144 = 360$

$2z + 288 = 360$

$2z = 360 - 288$

$2z = 72$

$\boxed{z = 36}$

$\angle A = x+y+z$

$144 = 54 + y + 36$

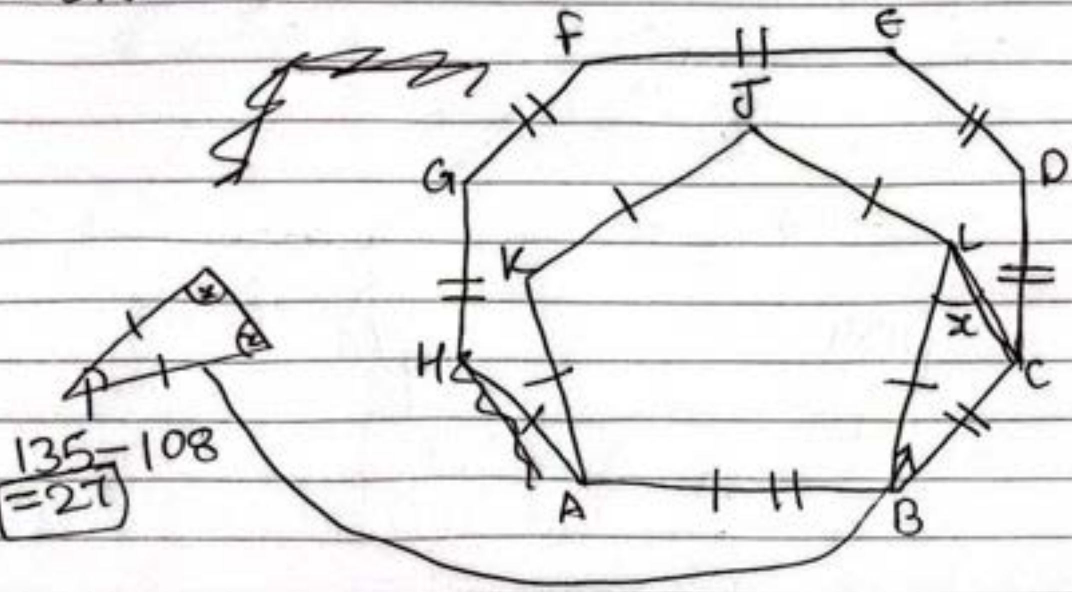
$144 = 90 + y$

$y = 144 - 90$

$\boxed{y = 54}$

$x = y$
$54 = 54$

Q- regular octagon ABCDGFGH and regular pentagon ABIJK



$\frac{(5-2)180}{5} = 108 = \angle ABI$

$\frac{(8-2)180}{8} = 135 = \angle ABC$

$\angle BCI = 135 - 108$
$= 27$

$x = \frac{180 - 27}{2} = \boxed{76.5}$

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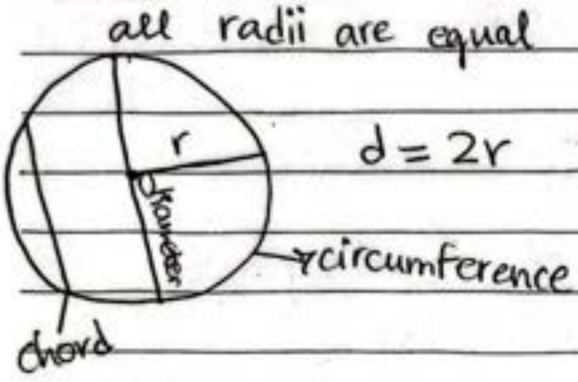
Circles:

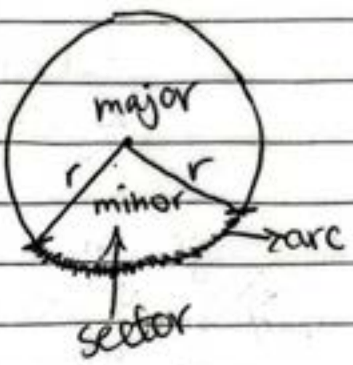
Angles in circles

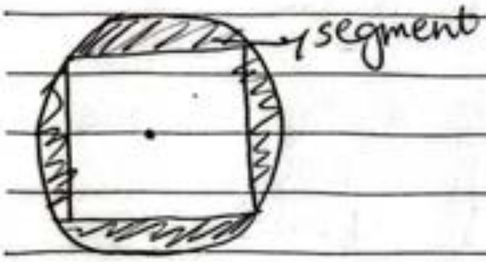
set of points that are equidistant from a fixed point.

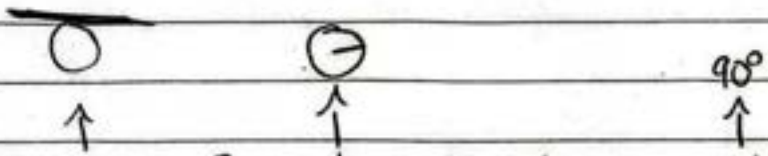
all radii are equal



$d = 2r$ $r = \frac{1}{2}d$







1) The angle between tangent & radius is always right.

2) The tangents that are drawn from a point outside the circle are equal.

$a+c=180$
$b+d=180$

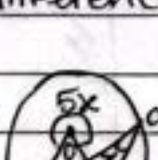
a quadrilateral inside a circle

3) The opposite angles in a cyclic quadrilateral add up to 180°

$x = 2y$ $y = \frac{1}{2}x$

4) The angle at the center is twice the angle at the circumference subtended by the same arc.

Q-



find measure of angle <AOB?

$2x$, $AOB = 2 \times ACB$
$AOB = 2 \times x$
$AOB = 2x$

$5x + 2x = 360$
$\frac{7x}{7} = \frac{360}{7}$
$x = 51.43°$

$AOB = 2 \times 51.43$
$AOB = 103$

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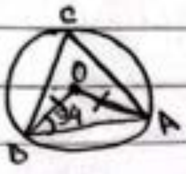
Q–



find <ACB

~~180~~

<BOA = 180 − (34+34)

< BOA = 112

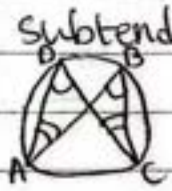
$<BCA = \frac{112}{2} = \boxed{56}$

5) The angle in a semi circle is always right

*has to touch diameter

*has to touch circumference

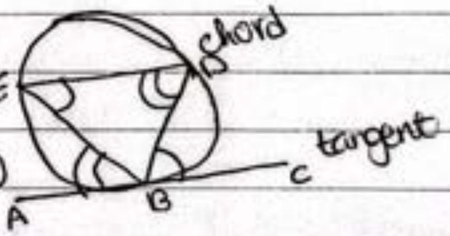
6) Angles at the circumference subtended by the same arc are equal in size.



<ADC = <ABC

<DAB = <DCD

7) Alternate segment theorem



< CBD = <BED

Q–

<CBE = <

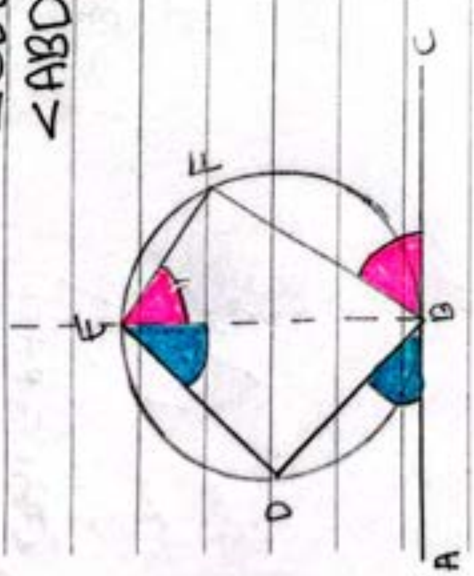
<ABD = <

<F

<A

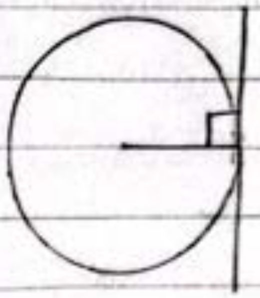
(draw

fir

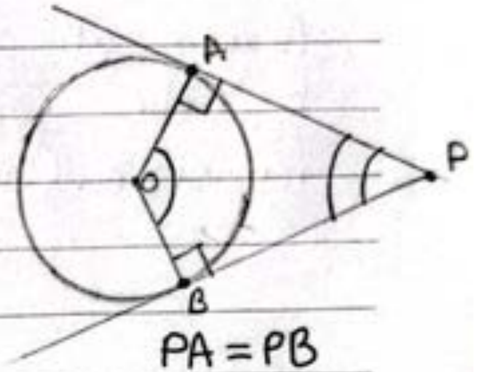


Angles in Circles:

- The angle between tangent and radius is always right.



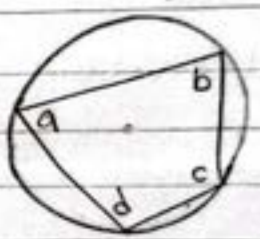
- The tangents that are drawn from a point outside are equal.



PA = PB

AOBP is kite

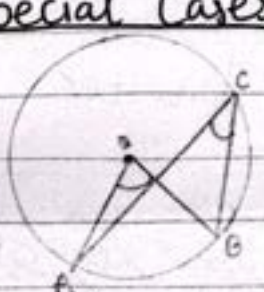
- The opposite angles in a cyclic quadrilateral add up to 180°. (all touch circumference)



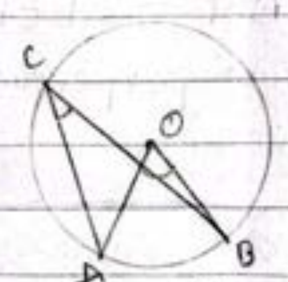
a + c = 180
b + d = 180

- The angle at the center is twice the angle at circumference subtended by the same arc.

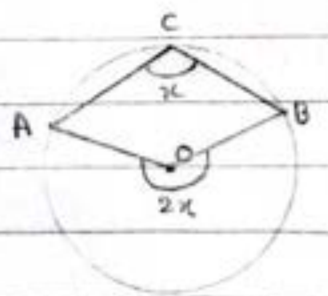
Special Cases:



<ABB = 1/2 x <AOB
<AOB = 2 x <ABB

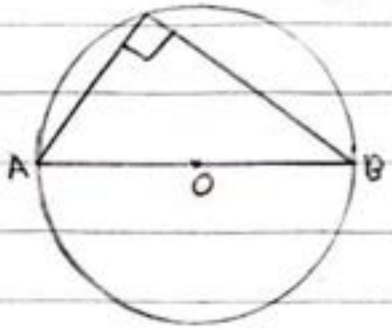


<ACB = 1/2 x <AOB
< AOB = 2 x <ACB



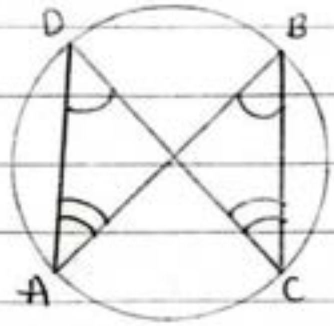
reflex < AOB = 2 x <ACB
<ACB = ~~<ACB~~ 2 x reflex <AOB

- The angle in a semi-circle is always right.



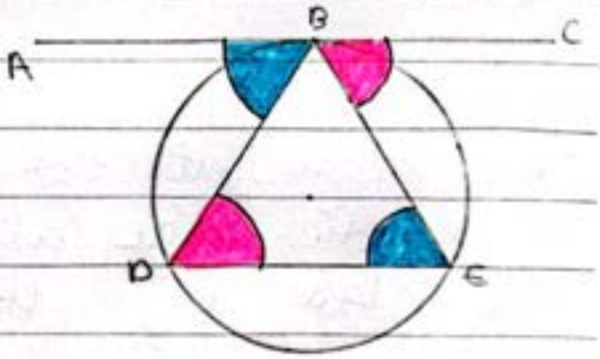
- Angles at circumference subtended by the same arc are equal in size. (same segment).

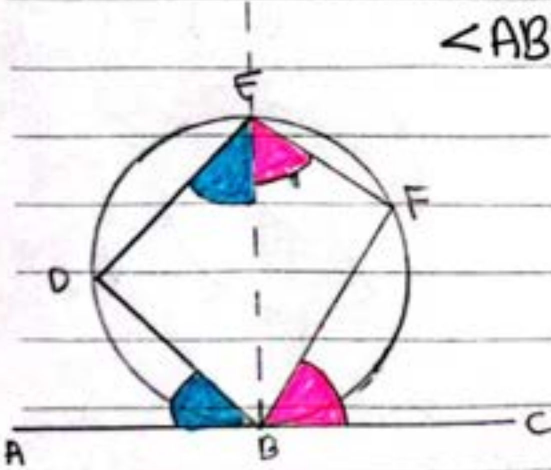
<DAB = <DCB
<ADC = <ABC



- Alternate segment theorem.

<CBE = <BDE
<ABD = <BED





<FBC = <BEF
<ABD = <BED
(draw perpendicular bisector first).

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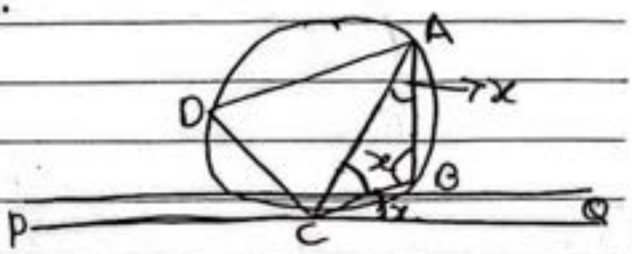
Angles and Circles

~~A, B, C and D are points on a circle.~~
~~TDI is the tangent to the circle D.~~
~~AB=AD~~
~~Angle ADI = [illegible]°~~

ABC and D are points on a circle
PCQ is a tangent to the circle.
$AB = CB$
Angle $BCQ = x^\circ$
Prove that angle $CDA = 2x^\circ$
Give reasons for each stage.



Draw line $\overrightarrow{AC}$

$\angle BAC = \angle BCQ = x^\circ$ (alternate segment)
$BA = BC$
$\angle BAC = \angle BCA = x^\circ$ (isosceles Δ)

$\angle ABC = 180 - 2x$
$\angle ADC = 180 \ominus (180 - 2x)$ (opposite angles
$\angle ADC = 180 - 180 + 2x$ in a cyclic
$\angle ADC = 2x$ quadrilateral).

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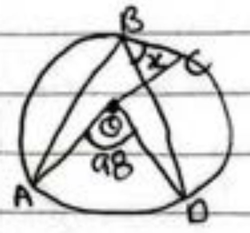
Top 10 - Talreem
Top 3 - prizes
Mock on 21st on All Algebra
All Numbers

Sub:
Date:

A, B, C and D are points on a circle, centre O.
ADC is diameter of the circle
Angle AOD = 98°
Workout the size of angle DBC.
Give reasons.



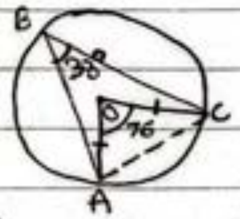
$\angle ABD = \frac{98}{2} = 49°$ (angle at the center is double at circumference).

$\angle ABC = 90°$ (angle opposite to diameter is 90°) (at Semi circle)

*We can connect dots to use other rules.

$\angle DBC = 90 - 49 = 41° = x$

Q - A, B & C are points on a circle, centre O.
Angle ABC = 38°
Workout the size of angle OAC.
Give reasons.



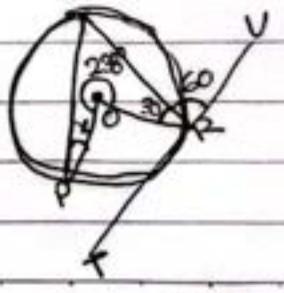
centre = 38×2 = 76° (angle at the centre is double at the circumference)

$\angle OAC = \angle OCA$

$\angle OAC = 180 - 76 = 104°$

$\angle OAC = \frac{104}{2} = \boxed{52°}$ (~~angles~~ isoscles △)

Q - P, Q and R are points on a circle, centre O.
TRV is the tangent to the circle at R.
Reflex angle POR = 238°
Angle QRV = 60°
Calculate the size of angle OPQ.
Give reasons.



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$360 - 238 = 122°$ ∠POR (angles at a point)

$\frac{122}{2} = 61°$ ∠PQR (angle at centre is 2x circumference)

∠ORV = 90° (angle between tangent and radius)

∠ORQ = 90 − 60 = 30°

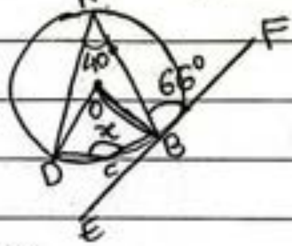
∠QPQ = 360 − [61 + 238 + 30]

= 31° (angles in quadrilateral = 360)

Q – A, B, C & D are points on a circle, centre O

EBF is tangent at B.

Workout ∠DCB



~~∠DOB = 40×2 = 80°~~

∠DCB = 180 − 40 = 140° = x

(opposite angles in cyclic quadrilateral add up to 180°)

Q-find ∠DOB

∠ADC = 90° ξ

∠OBF = 90° (between tangent)

∠OBA = 90 − 66 = 24° ∠DOB = 40×2 = 80

∠ADO = 360 − (24 + 280 + 40) reflex ∠DOB = 360 − 80 = $\boxed{280°}$

= $\boxed{16°}$

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Q -

A&C are points on a circle, centre O
DA is the tangent to the circle A and DC is the tangent to the circle at C.
Angle ADC = 48°
Workout reflex angle = AOC

$\angle ADC = 48^\circ$

~~$\angle AOC = 48 \times 2 = 96^\circ$~~

$\angle DAO = 90^\circ$

$\angle DCO = 90^\circ$

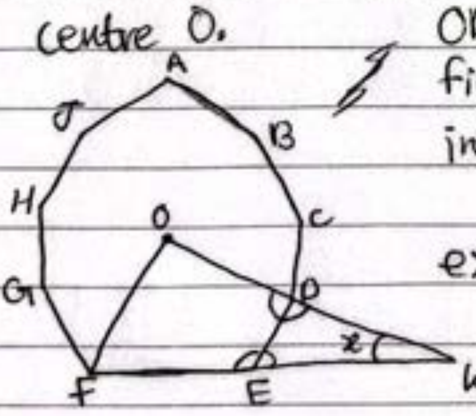
$\angle AOC = 360 - (90 + 90 + 48)$

$= 132^\circ$

reflex $= 360 - 132$

$\boxed{= 228^\circ}$

Q - Here is a 9-sided regular polygon ABCDEFGHIJ, with centre O.



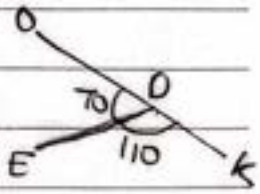
ODK & FEK are straight lines
find x.

interior $= 180 - \frac{360}{9} = 140^\circ$

exterior $= 40^\circ$

$\angle EDK = 180 - 70 = 110^\circ$

$x = 180 - (110 + 40) = 30^\circ$



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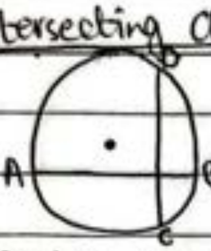
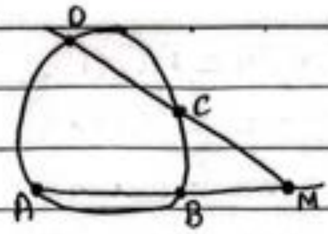
Intersecting Chords:

①



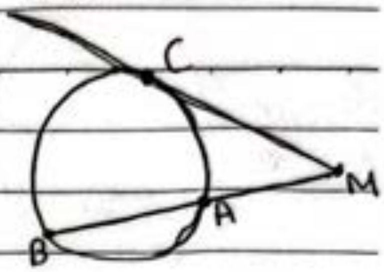
$MB \times MA = MC \times MD$

②



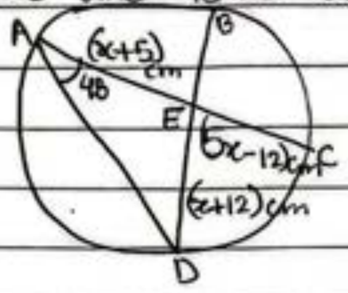
$MC \times MD = MB \times MA$

③



$(MC)^2 = MA \times MB$

Q- ABC and BED are chords of a circle.



$AE = (x+5)cm$

$BE = x cm$

$CE = (5x-12)cm$

$DE = (x+12)cm$

Angle $DAE = 48°$

$\boxed{\text{find } \angle ADE}$

$EC \times EA = EB \times ED$

$(5x-12)(x+5) = x(x+12)$

$5x^2+25x-12x-60 = x^2+12x$

$5x^2+13x-60-x^2-12x=0$

$4x^2+x-60=0$

$(4x-15)(x+4)=0$

$\boxed{x=\frac{15}{4}}$, $\cancel{x=-4}$

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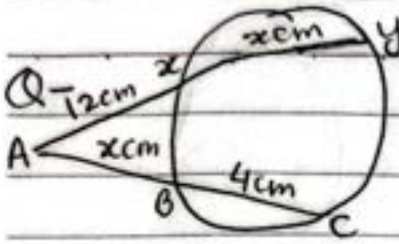


$$AX \times AY = AB \times AC$$

$$12[12+x] = x[4+x]$$

$$144+12x = 4x+x^2$$

$$x^2+4x-12x-144=0$$

$$x^2-8x-144=0$$

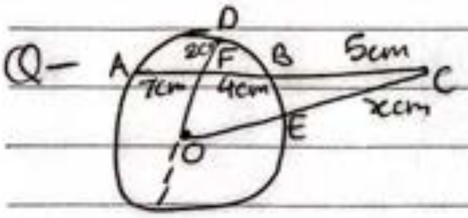
$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-144)}}{2(1)}$$

$$x=\frac{8\pm8\sqrt{10}}{2}$$

$$AC = x+4$$

$$AC = 16.6+4$$

$$AC = 20.6cm$$



find x. DO=ycm

$$DF \times FZ = FB \times FA$$

$$2(y-2) = 4\times7$$

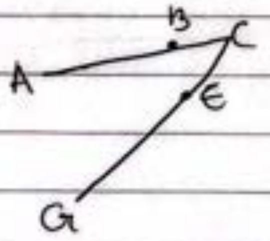
$$2y-4 = 28$$

$$2y = 32$$

$$y = 16$$

$$DO = 16$$

$$DZ = 32$$



$$BC \times AC = EC \times GC$$

$5\times16 =$ [illegible] $x(x+32)$

$$80 = x^2+32x$$

$$x^2+32x-80=0$$

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yes

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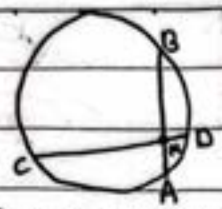
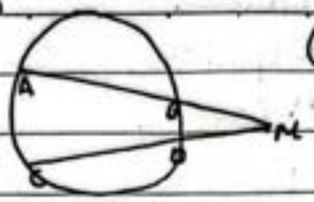
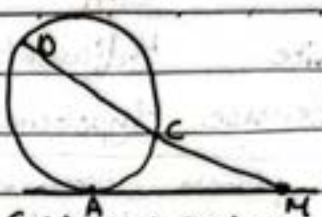
Continue Intersecting Chords, lengths

①


$MD \times MC = MA \times MB$

②


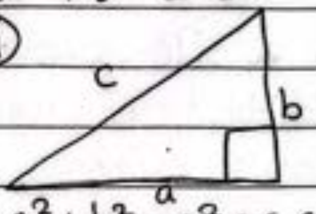
$MB \times MA = MD \times MC$

③


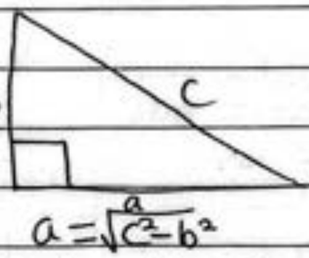
$(MA)^2 = MC \times MD$

Pythagoras theorem : (right angle triangle only)

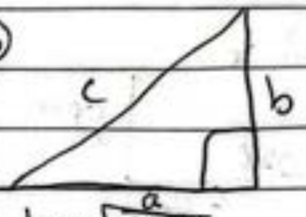
$a^2+b^2=c^2$

①


$a^2+b^2=c^2$ or $c=\sqrt{b^2+a^2}$

②


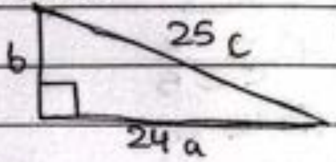
$a=\sqrt{c^2-b^2}$

③


$b=\sqrt{c^2-a^2}$

* biggest angle is opposite to biggest length and forth.

ex: find



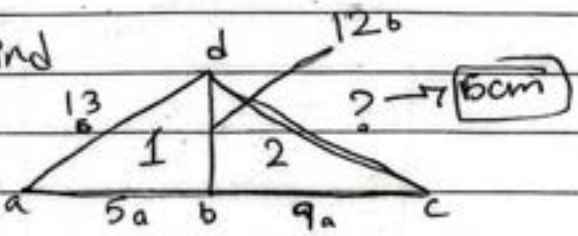
$b=\sqrt{c^2-a^2}$

$b=\sqrt{25^2-24^2}$

$b=\sqrt{49}$

$b=7cm$

find



1) $b=\sqrt{c^2-a^2}$

$b=\sqrt{13^2-5^2}$

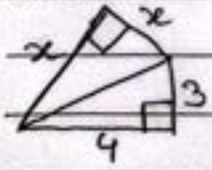
$b=12$

2) $c=\sqrt{a^2+b^2}$

$c=\sqrt{9^2+12^2}$

$c=15$

find



find x

$c=\sqrt{a^2+b^2}$

$c=\sqrt{4^2+3^2}$

$c=\sqrt{25}$

$c=5$

$c^2=x^2+x^2$

$c^2=2x^2$

$5^2=2x^2$

$25=2x^2$

$x^2=12.5$

$x=\sqrt{12.5}$

$x=3.54$

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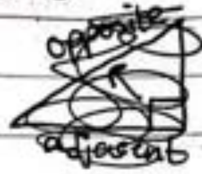
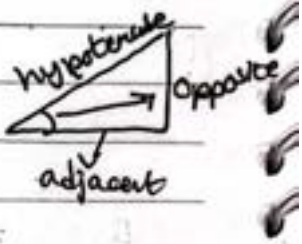
Sin, Cos & Tan: (only acute angles and right angles)

Sine	C~~AH~~osine	Tangent
Opposite	Adjacent	Opposite
Hypotenuse	Hypotenuse	Adjacent

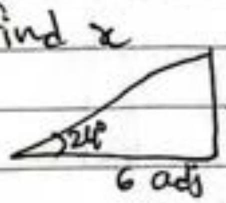




these are ratios to the lengths

$\sin x = \frac{\text{opposite}}{\text{adjacent}}$ | $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$ | $\tan x = \frac{\text{opposite}}{\text{adjacent}}$

find x



$\tan 24 = \frac{x}{6}$

$\tan 24 \times 6 = x$

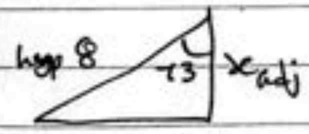
$2.375 = x$

$2.38 \approx x$

*make sure correct calc settings

*to find a side length:
- we need to have 1 angle
- 1 side

find x



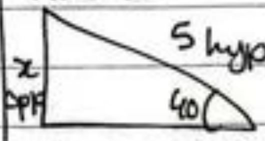
$\cos 73 = \frac{x}{8}$

$8 \times \cos 73 = x$

$3.292 = x$

$3.29 \approx x$

find x



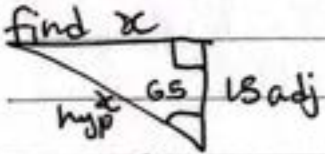
$\sin 40 = \frac{x}{5}$

$5 \times \sin 40 = x$

$2.939 = x$

$2.94 \approx x$

find x

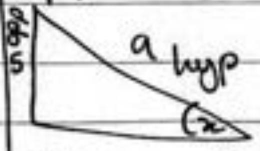


$\cos 65 = \frac{15}{x}$

$x = \frac{15}{\cos 65}$

$x = 28.7$

find x

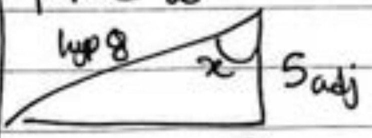


$\sin x = \frac{5}{9}$

$x = \sin^{-1}(5/9)$

$x = 33.75$

find x



$\cos x = \frac{5}{8}$

$x = \cos^{-1}(5/8)$

$x = 51.32$

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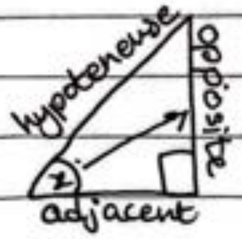
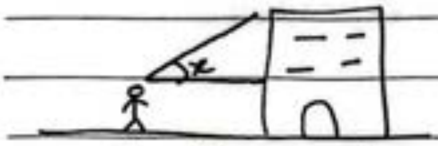
Trignometry

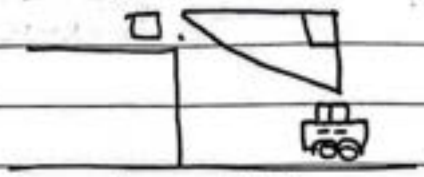
SOH CAH TOA (applied on acute angles in right angle triangles)



Angle of elevation & depression : (based on SOHCAHTOA)



angle of elevation



angle of depression

ex:

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ometry.

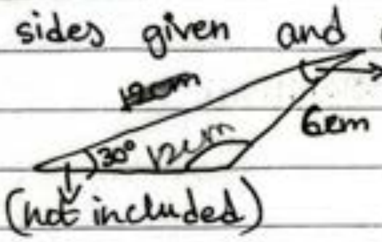
Sin and Cosine rules.

*acute and obtuse triangles. ONLY

*finding missing sides and missing lengths.

Sine rule:

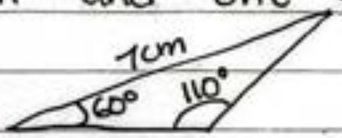
① 2 sides given and one not included angle.



named (included angle)

between 2 sides

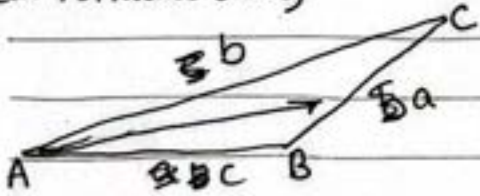
② 2 angles given and one side



(we use only 2 depending on the question)

formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\boxed{\text{or}}$ $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(in formula sheet)

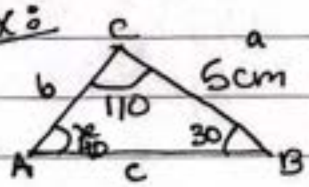


uppercase letters are ~~angels~~ angles.

lowercase letters are lengths.

to get lower case, whatever is opposite to the upper case angle gives you lower case length.

ex:



find $\overline{AC}$?

$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$

(because we have a and we want to find b meaning they are both involved).

$x = 180 - (110 + 30)$

$x = 40°$

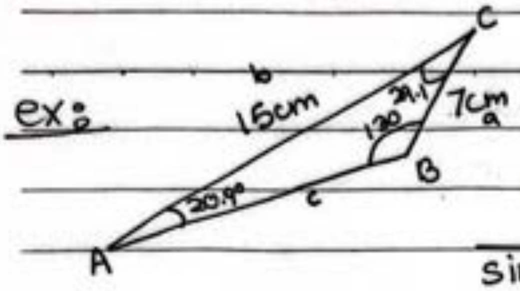
$\frac{5}{\sin 40} = \frac{b}{\sin 30}$

$\frac{5 \times \sin 30}{\sin 40} = b$

$3.89 \approx b$

Sub:
Date:

ex:



find $\overline{AB}$?

*you cant find a side until you know its angle.

$$\frac{7}{\sin A} = \frac{15}{\sin 130}$$

$$~~\sin A = \frac{15}{7x\sin 130}~~ \qquad \sin A = \frac{7x\sin 130}{15}$$

$$A = \sin^{-1}\left(\frac{7x\sin 130}{15}\right)$$

$$A = 20.9$$

$$C = 180-(130+20.9)$$
$$C = 29.1$$

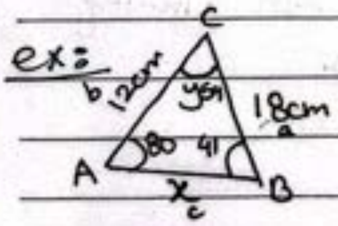
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{15}{\sin 130} = \frac{c}{\sin 29.1}$$

$$\frac{\sin 29.1 \times 15}{\sin 130} = c$$

$$9.52 \approx c$$
$$9.52 = AB$$

ex:



find $\overline{AB}$?

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{18}{\sin 80} = \frac{12}{\sin B}$$

$$\frac{12x\sin 80}{18} = \sin B$$

$$B = \sin^{-1}\left(\frac{12x\sin 80}{18}\right)$$
$$B = 41.0$$
$$C = 180-(80+41)$$
$$C = 59°$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

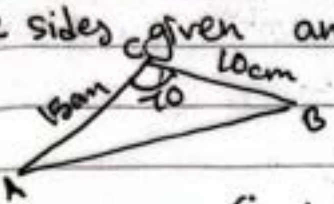
$$\frac{18}{\sin 80} = \frac{c}{\sin 59}$$

$$\frac{\cancel{\sin 80}\,18x\sin 59}{\sin 80} = c$$

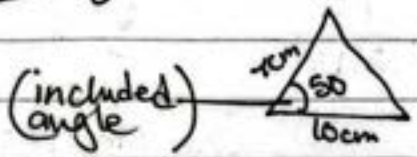
$$15.7 = c$$
$$\boxed{15.7 = AB}$$

[illegible] rule:

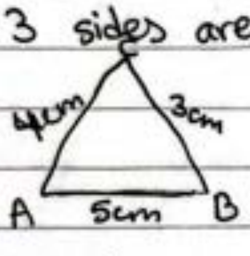
① 2 sides given and one included angle.



(Find missing sides)



② 3 sides are given



(find missing angles)

formula:

[to get sides] missing is subject

$a^2 = b^2 + c^2 - 2bc \cos A$ (if you are trying to get a)

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = b^2 + a^2 - 2ab \cos C$

[to get angles] missing is subject

$\cos A = \frac{c^2 + b^2 - a^2}{2cb}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

* dont forget to square root answer when finding sides, and get $\cos^{-1}$ when getting angles.

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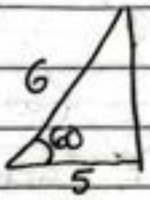
area of triangle :

$\text{area} = \frac{1}{2} \times b \times h$

$$\text{area} = \frac{1}{2} \times \underbrace{a \times b}_{\text{product of any 2 sides}} \times \overbrace{\sin C}^{\text{sin of included angle (sandwiched)}}$$



$\frac{1}{2} \times 6 \times 5 \times \sin 60$

$15 \times \sin 60$

12.99

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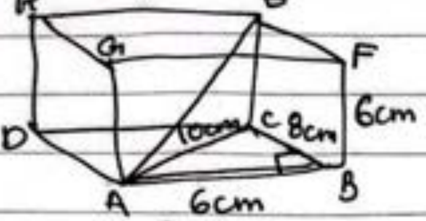
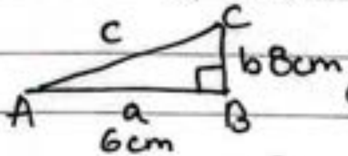
Bearings & ~~3rd~~ 3d trignometry.

3d trignometry:

Q –

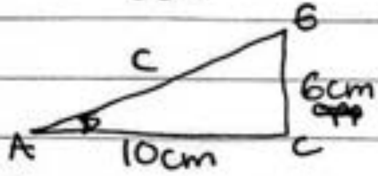


find the angle between $\overline{AG}$ and the plan ABCD?
the base

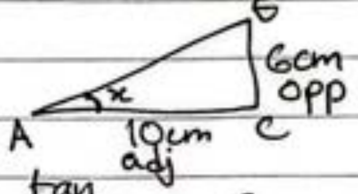


$c=\sqrt{a^2+b^2}$
c= 10cm

$\overline{AC} = 10cm$
$\overline{AE} = 11.7cm$



$c=\sqrt{a^2+b^2}$
c= 11.66
c≈11.7cm
X



$\overset{tan}{\cancel{sin}}\, x = \frac{6}{10}$

$x = \overset{tan}{\cancel{sin}}{}^{-1}\left(\frac{6}{10}\right)$

30.96

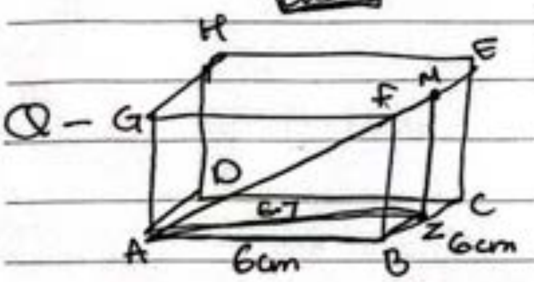
x = ~~30.9637565~~

x ≈ 31°

cube

all sides are equal

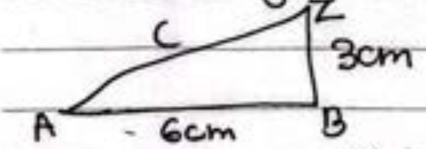
Q –



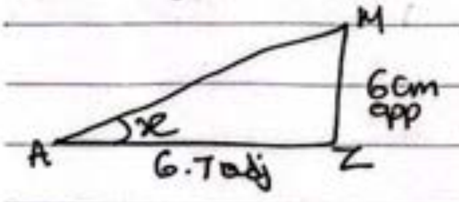
find the size of the angle between $\overline{AM}$ and the base ABCD?

*if no sides, we can ~~use~~ assume numbers or use unknowns

let one edge be 6cm



$c=\sqrt{6^2+3^2}$
c= 6.7



$c=\sqrt{6.7^2+6^2}$
c= 8.99cm
X

$\tan x = \frac{6}{6.7}$

$x = \tan^{-1}\left(\frac{6}{6.7}\right)$

$\boxed{x = 41.8^\circ}$

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Sub:
Date:

Group 2: sun: 8:45 wed: 8:45

sunday 21/1 off
wednesday (mock) 24/1
friday/saturday 26/27/1 off

return 28/1

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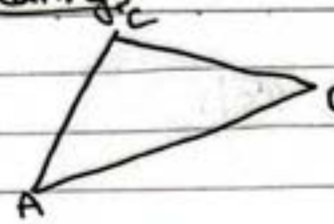
Date: 31/1/2024

Sine & Cosine Rules

Bearings



<CAB is obtuse

Answer

<CAB = 58° (acute)

180 − 58 = $\boxed{122°}$

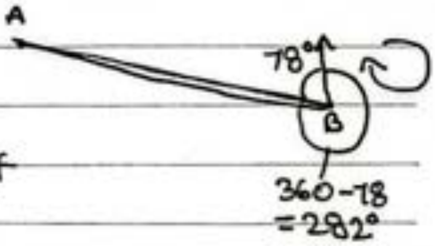
Convert from acute to obtuse
= 180 − acute = obtuse

Convert from obtuse to acute
= 180 − obtuse = acute

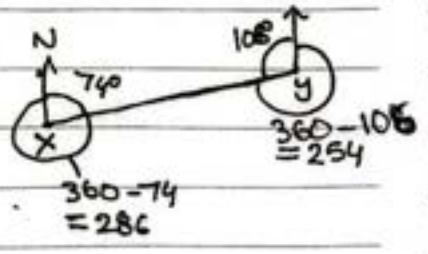
Location

Bearing : Observations

1) Measure Clockwise direction
2) Measure from the north
3) Write ans in 3 digits form



find bearing of A from B
find bearing of B from A
center of bearing



find bearing of Y from X = 74 centre of bearing
find bearing of X from Y ~~286~~

measure clockwise direction, from north, 3 digits.

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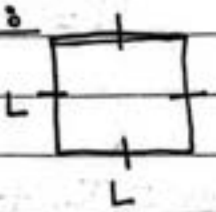
Date: 2/2/2024

Area & perimeter of 3d Shapes

Mensuration:

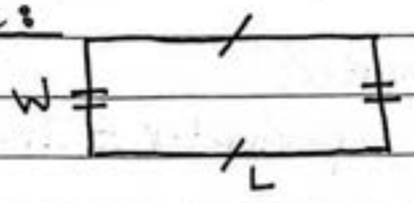
square:



all sides are equal

area = length × length , perimeter = 4 × length

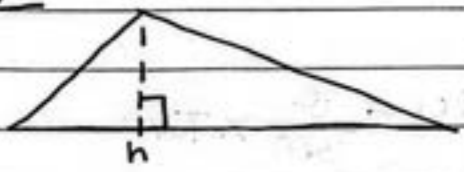
rectangle:



area = length × width

perimeter = (2 × length) + (2 × width)

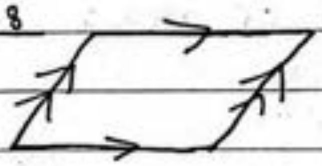
triangle:



$\text{area} = 1/2 \times \overbrace{b}^{\text{base}} \times \overbrace{h}^{\text{height}}$, $\text{area} = 1/2 \times \overbrace{a \times b}^{\text{any 2 sides}} \times \overbrace{\sin}^{\text{Sand with angle}}$

perimeter = add all sides.

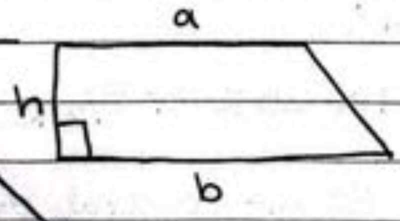
parallelogram:



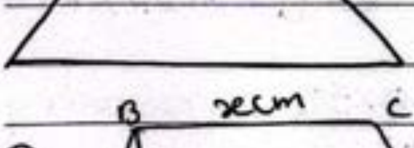
area = base × height, area = a × b × sin

perimeter = add all sides

trapezium:

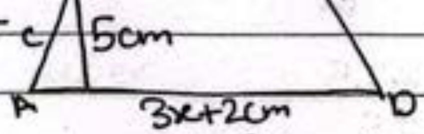


$\text{area} = \frac{a+b}{2} \times h$, $1/2(a+b)h$



perimeter = adding all sides

Q-



area = 100 cm²

find perimeter?

a+

~~$(3x+2) - x = \frac{2x+2}{2} = x+1$~~

$\text{area} = 1/2 \times b \times h$

$\text{area} = 1/2 \times (x+3x+2) \times 5 =$ ~~$\sqrt{5^2+(x+1)^2} = \sqrt{x^2+2x+26}$~~

$\text{area} = 1/2 \times 4x+2 \times 5$ ~~$\text{perimeter} = (3x+2) + x + (\sqrt{x^2+2x+26}) +$~~

$100 = 1/2 \times 4x + 2 \times 5$

$\frac{100-2}{1/2 \times 5} = 4x$, $\frac{98}{2.5} = 4x$, $\frac{39.2}{4} = \frac{4x}{4}$, $9.\overset{5}{8} = x$

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arc length :

(L)

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$

if we have arc or θ:

$$\frac{\theta}{360} = \frac{\text{arc length}}{2\pi r}$$

perimeter of sector:

$$p = 2r + L$$

Q- a sector has a radius 5cm and central angle 60, find arc length.

$$\text{arc} = \frac{60}{360} \times 2\pi 5 = 1/6 \times 10\pi = 5/3\,\pi\,\text{cm}$$

Q- arc length = 12cm, central angle is 82, find area and perimeter of sector.

$$\cancel{\text{area}=}\ \frac{82}{360} = \frac{\overset{12}{\cancel{\text{arc}}}}{2\pi r} = 2\pi r = \frac{360 \times 12}{82}$$

$$2\pi r = \frac{2160}{41}$$

$$r = \frac{2160/41}{2\pi}$$

$$r = 8.38\,\text{cm}$$

$$\text{area} = \frac{82}{360} \times \pi\, 8.38^2$$

$$\text{area} = 50.3\,\text{cm}^2$$

$$\text{perimeter} = 2r + L$$
$$= 2(8.38) + 12$$

$$\text{perimeber} = 28.76\,\text{cm}$$

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Sub:

Date:

arc length :

(L)

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$\text{area} = \frac{82}{360} \times \pi\, 8.38^2$

$\text{area} = 50.3\text{ cm}^2$

$2\pi r = \frac{2160}{41}$

$r = \frac{2160/41}{2\pi}$

$r = 8.38\text{cm}$

$\text{perimeter} = 2r + L$

$= 2(8.38) + 12$

$\text{perimeter} = 28.76\text{ cm}$

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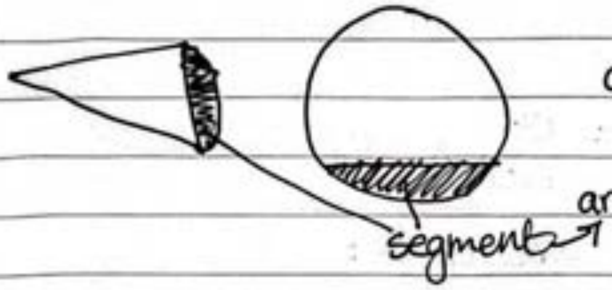
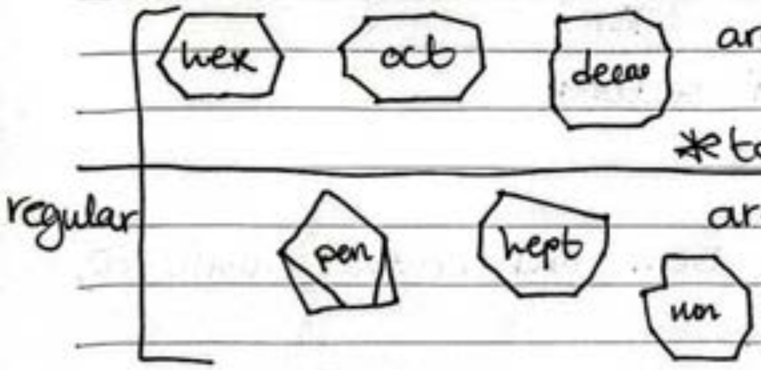
Shaded Segments, Area of polygons



area = area of sector - area of triangle.

$\text{area} = \left(\frac{\theta}{360} \times \pi r^2\right) - \left(\frac{1}{2} ab\sin C\right)$



area of hexagon = $\underbrace{6}_{\text{no. of sides}}$ x area of triangle

*to get angle, 360 divide by no. of side

area of polygon = $\underbrace{5}_{\text{no. of sides}}$ x area of triangle

*if no side, we split and do pythagoras.

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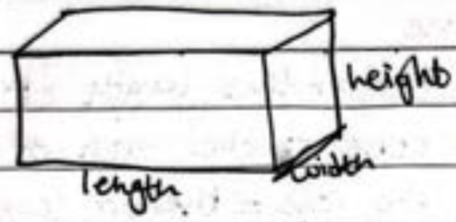
Date: 9/2/2024

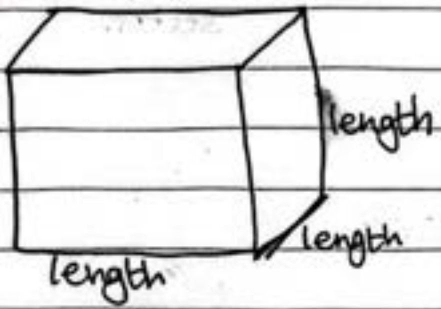
Surface area and Volume



cuboid:

volume of cuboid = length x width x height.

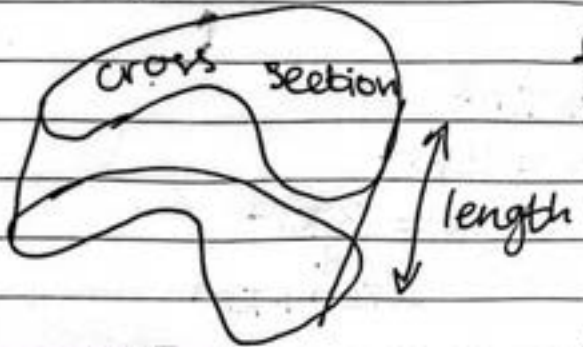
total surface area = 2[LW+LH+WH]



cube:

volume = length^3

total surface area = $6L^2$



prism:

volume = area of cross section x length

surface area = sum of area of all sides.

cylinder:

volume = area of cross section x length

$= \pi r^2 \times h$

total surface area = $2\pi rh + 2\pi r^2$

curved surface area = $2\pi rh$

cone:

volume = 1/3 x base area x height

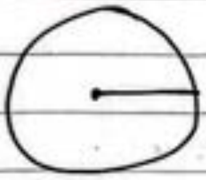
$= \frac{1}{3} \times \pi r^2 h$

total surface area = $\pi r l$ (L is slanted height

curved surface area = $\pi r^2 + \pi r l$

Cone → sector & sector → cone

1) The slant of the cone = ~~sector~~ the radius length of sector
2) The curved surface area of the cone = the area of sector
3) The circumference of the base of the cone = the arc length of sector.



sphere :

volume = $4/3\pi r^3$
surface area = $4\pi r^2$

hemisphere :

volume = $2/3\pi r^3$
curved surface area = $2\pi r^2$
total surface area = $3\pi r^2$

$$\downarrow$$

$$2\pi r^2+\pi r^2$$

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Date: 11/2/2024

✱ Continue Mensuration 3D Shapes

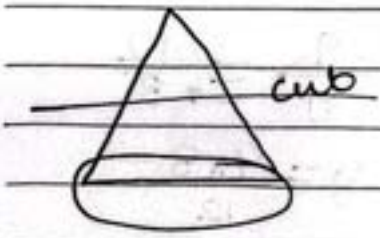
combined 3d shapes :

-total surface area of combined shapes is whatever you can see.

pyramid :

$\text{volume} = 1/3 \times \overset{\text{base area}}{b} \times h$

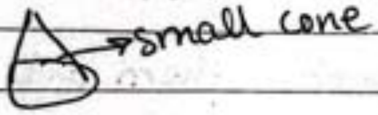
surface area = sum of all areas of all faces.

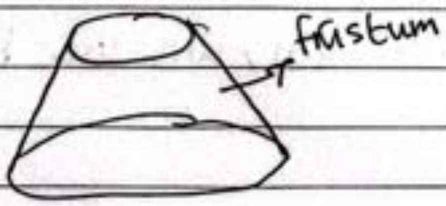


frustum : (chopped cone)

volume = volume of large cone - volume of small cone

~~volume = 1/3π~~





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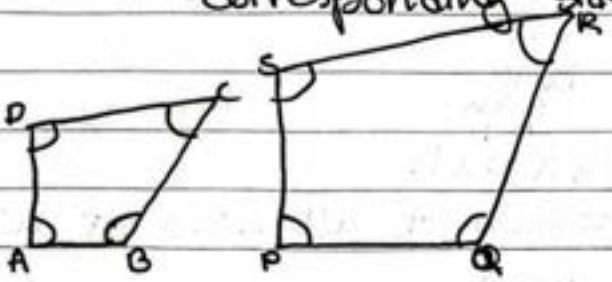
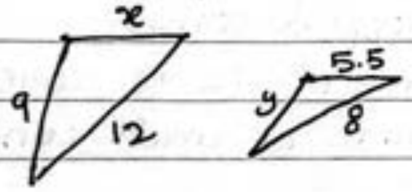
Similarity

*To say that ~~sim~~ shapes are similar :
- corresponding angles are equal
- corresponding sides are in the same ratio



$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RS}{CD} = \frac{SP}{DA}$$

Q- the triangles are similar. Find x and y



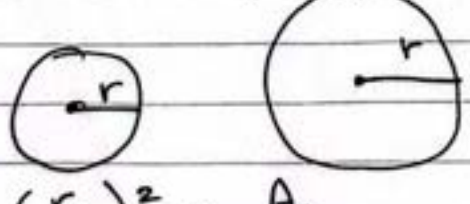
$$\frac{x}{5.5} = \frac{12}{8}$$
$$x = 5.5 \times \frac{12}{8}$$
$$x = 8.25$$

$$\frac{9}{y} = \frac{12}{8}$$
$$y = \frac{9 \times 8}{12}$$
$$y = 6$$

*if we have 2 shapes, we separate them and use ratio.

*in the ratio, if we start with big, we use big too & vice versa.

2 similar shapes (area):



*circle A has radius 5cm, area is 20cm². circle B has area 80cm², find radius length.

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{A_1}{A_2}$$

$$\sqrt{\left(\frac{5}{x}\right)^2} = \sqrt{\frac{20}{80}}$$

$$\frac{5}{x} = \sqrt{\frac{20}{80}} = \frac{5}{x} = \frac{1}{2}$$

~~$x = \sqrt{80 \times 5}$~~ ~~$x = 2\sqrt{5}$ cm~~ $x = \frac{5 \times 2}{1}$

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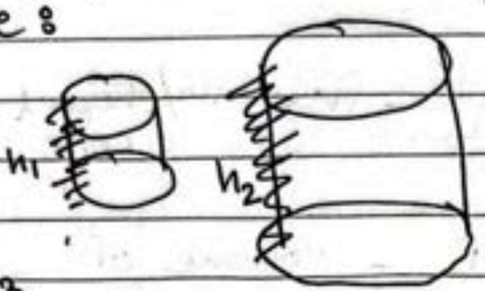
volume :



Q- cone A has height 5 cm and volume 500 cm³.
cone B has height 8cm
find its volume

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{V_1}{V_2}$$

~~$\left(\frac{5}{8}\right)^3 = \frac{500}{x}$~~

$$\left(\frac{5}{8}\right)^3 = \frac{500}{x}$$

~~$x = 500 \times \left(\frac{5}{8}\right)^3$~~

$$\frac{5^3}{8^3} = \frac{500}{x}$$

~~$x = 122\,cm^3$~~

$$\frac{500 \times 8^3}{5^3} = x$$

$$x = 2048\,cm^3$$

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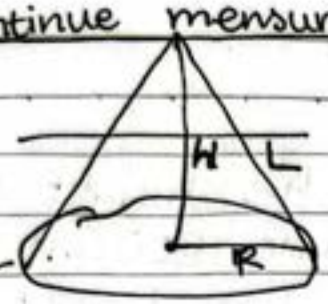
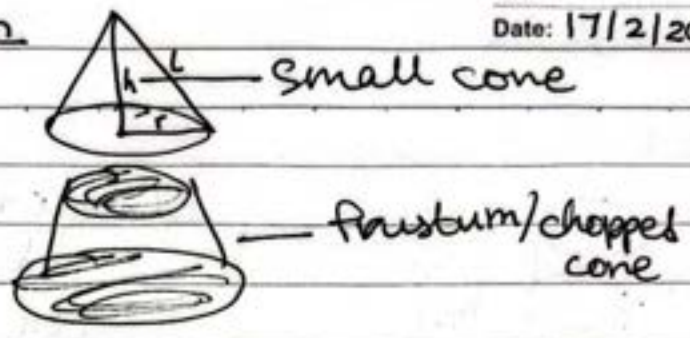
Continue mensuration





volume frustum = volume of large cone − volume of small cone

$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$\boxed{\text{volume} = \frac{1}{3}\pi r^2 h}$

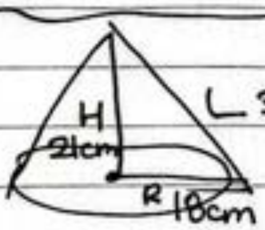
$$\boxed{\frac{r}{R} = \frac{h}{H} = \frac{l}{L}}$$

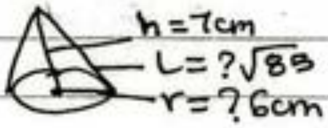
curved surface area = C.S.A. (large cone) − C.S.A. (small cone)

$\pi RL - \pi rl + \underbrace{\pi r^2 + \pi R^2}$

depending on ~~which face was~~ closed

Q-





Find the slanted height for both cones and volume of frustum.

$\frac{r}{18} = \frac{7}{21}$

$r = \frac{18 \times 7}{21} = 6cm$

$L = \sqrt{7^2 + 6^2}$

$L = \sqrt{85}$ cm

$L = \sqrt{21^2 + 18^2}$

$L = 3\sqrt{85}$ cm

volume of frustum

$= \frac{1}{3}\pi 18^2 \times 21 - \frac{1}{3}\pi 6^2 \times 7$

$= 2184\pi$

$\boxed{\approx 6861 cm^3}$

areas:

$$\left[\frac{r}{R}\right]^2 = \frac{A_1}{A_2} \text{ or } \left[\frac{h}{H}\right]^2 = \frac{A_1}{A_2} \text{ or } \left[\frac{l}{L}\right]^2 = \frac{A_1}{A_2}$$

example :

2 similar cones ; the smaller cones has a curved surface area of 50cm² the ratio between their radii is 2:3 . find c.s.a of larger cone.

$\left[\frac{r}{R}\right]^2 = \frac{A_1}{A_2}$ r : R
2 : 3

$\left[\frac{2}{3}\right]^2 = \frac{50}{A_2}$

$\frac{4}{9} = \frac{50}{A_2}$

$\frac{9 \times 50}{4} = A_2$

$\underline{112.5cm^2 = A_2}$

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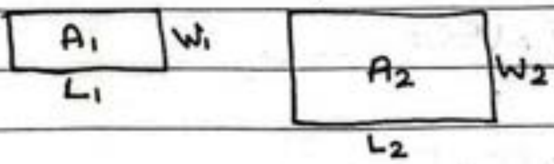
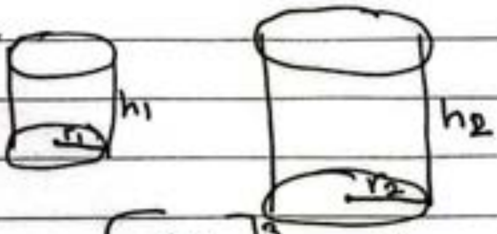
similar areas :



$\left[\frac{L_1}{L_2}\right]^2 = \frac{A_1}{A_2}$

similar volumes :



~~$\left[\frac{V_1}{V_2}\right]^3 = \frac{A_1}{A_2}$~~

$\left[\frac{r_1}{r_2}\right]^3 = \frac{V_1}{V_2}$ or $\left[\frac{h_1}{h_2}\right]^3 = \frac{V_1}{V_2}$

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Q - 2 similar cylinders, the surface area of the smaller to the surface area of larger is 4:9.

a) find the volume of the large cylinder if the volume of the small cylinder is 1500cm³

$$\left[\frac{h_1}{h_2}\right]^2 = \frac{A_1}{A_2} \longrightarrow \sqrt{\left[\frac{h_1}{h_2}\right]^2} = \sqrt{\frac{4}{9}}$$

$$\boxed{\frac{h^1}{h_2^2} = \frac{2}{3}}$$

$$\left[\frac{h_1}{h_2}\right]^3 = \frac{V_1}{V_2} \longrightarrow \left[\frac{2}{3}\right]^3 = \frac{1500}{V_2}$$

$$\frac{8}{27} = \frac{1500}{V_2}$$

$$\frac{27 \times 1500}{8} = V_2$$

$$\boxed{5062.5 = V_2}$$

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Continue Similarity

In similar shapes:

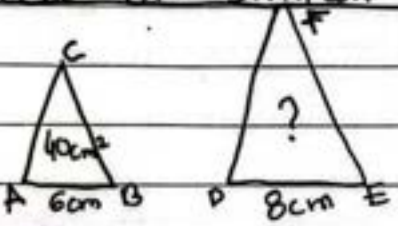
- corresponding angles are equal.
- corresponding sides are the same ratio.

*we can get ratio by triangle label

$\Delta ABC \sim \Delta XYZ$

$$\frac{AB}{XY} = \frac{AC}{XZ}$$

area of similar shapes:



$$\left[\frac{AB}{DE}\right]^2 = \frac{A_1}{A_2}$$

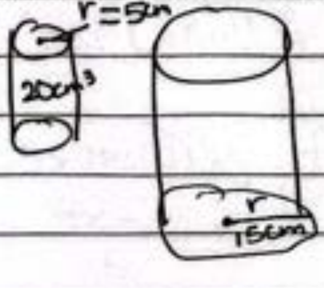
$$\left[\frac{6}{8}\right]^2 = \frac{40}{A_2}$$

$$\frac{36}{64} = \frac{40}{A_2}$$

$$A_2 = \frac{40 \times 64}{36}$$

$$A_2 = 71.1 \text{cm}^2$$

volume of similar shapes:



$$\left[\frac{r_1}{r_2}\right]^3 = \frac{V_1}{V_2} \qquad \left[\frac{h_1}{h_2}\right]^3 = \frac{V_1}{V_2}$$

$$\left[\frac{5}{15}\right]^3 = \frac{20}{V_2}$$

$$\frac{125}{3375} = \frac{20}{V_2}$$

$$V_2 = \frac{3375 \times 20}{125}$$

$$V_2 = 540 \text{cm}^3$$

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Sub:
Date:

area and volume of similars

$$\left[\frac{V_1}{V_2}\right]^2 = \left[\frac{A_1}{A_2}\right]^3$$

Vase A has volume 200cm^3 and surface area 80cm^2
Vase B has volume 800cm^3, find its surface area?

$$\left[\frac{200}{800}\right]^2 = \left[\frac{80}{A_2}\right]^3$$

$$\sqrt[3]{\frac{40000}{640000}} = \sqrt[3]{\left[\frac{80}{A_2}\right]^3}$$

$$\frac{80}{A_2} = \sqrt[3]{\frac{40000}{640000}}$$

$$\frac{80}{A_2} = 0.397$$

$$A_2 = \frac{80}{0.397}$$

$$A_2 \approx 201.587 \text{ cm}^2$$

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Functions

Sub:

Date: 21/2/2024

Objectives:

- evaluating functions
- composition of functions
- inverse functions (quadratic)
- domain and range

✱ blender

$\boxed{\text{input}} \longrightarrow \boxed{x\ {}^{-}\div{}^{+}\sqrt{\ }} \longrightarrow \boxed{\text{outpub}}$

orange   orange juice

$x \longrightarrow \boxed{{}^{-}x\ {}^{\prime}\div{}^{+}\sqrt{\ }} \longrightarrow \boxed{f(x)}$ or $\boxed{y}$ } function notation

$x \longrightarrow \boxed{\times 2 - 5 \div 3}\quad \frac{2x-5}{3} = f(x)$

$f(x) = \frac{3x-2}{5}$

find

a) $f(2) = \frac{3(2)-2}{5} = \frac{6-2}{5} = 0.8$

b) $f(-1) = \frac{3(-1)-2}{5} = \frac{-3-2}{5} = -1$

c) $f(3x-1) = \frac{3(3x-1)-2}{5} = \frac{9x-3-2}{5} = \frac{9x-5}{5}$

simplify function

$\frac{9x-3}{6} = \frac{\overset{1}{\cancel{3}}(3x-1)}{\underset{2}{\cancel{6}}} = \frac{3x-1}{2}$ ← common factor

find x when $f(x) = 3$ → means 3 replaces $f(x)$ in main formula

$\frac{3x-2}{5} = 3$

$3x-2 = 3 \times 5$

$3x = 15+2$

$x = \frac{17}{3}\ \boxed{= 5.67}$

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$f(x) = \frac{3x-2}{5+2x}$

find

a) $f(3) \rightarrow \frac{3(3)-2}{5+2(3)} = \frac{9-2}{5+6} = \frac{7}{11}$

b) $f(x-2) \rightarrow \frac{3(x-2)-2}{5+2(x-2)} = \frac{3x-6-2}{5+2x-4} = \frac{3x-8}{2x+1}$

c) find x, when $f(3x-1) = 4 = \frac{3(3x-1)-2}{5+2(3x-1)} = 4$

$= \frac{9x-3-2}{5+6x-2} = 4$

$= \frac{9x-5}{3+6x} = 4$

$= 9x-5 = 4(3+6x)$

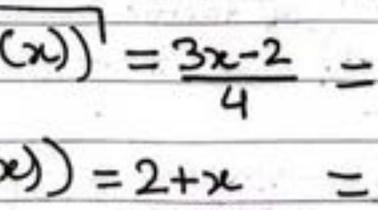
$= 9x-5 = 12+24x$

$= -12-5 = 24x-9x$

$= -17 = 15x$

$= \boxed{x = \cancel{\frac{15}{17}}\ \frac{-17}{15}}$

composite 8 ✗



$\rightarrow fg(x)$
$\rightarrow fg$
$\rightarrow f(g(x))$
same thing

$f(x) = \frac{3x-2}{4}$ $g(x) = 2+x$

input is 'g'

find:

1) $fg(x) = f(g(x)) = \frac{3x-2}{4} = \frac{3(2+x)-2}{4}$

2) $gf(x) = g(f(x)) = 2+x = \frac{6+3x-2}{4} = \boxed{\frac{3x+4}{4}}$

$= \frac{\overset{\times 4}{2}}{\times 4\ 1} + \frac{3x-2}{4}$ ~~$= 2 + \frac{3x-2}{4}$~~ ~~$= \frac{3x}{8}$~~

$= \frac{8}{4} + \frac{3x-2}{4}$

$= \frac{8+3x-2}{4} = \boxed{\frac{3x+6}{4}}$

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Sub:
Date:

$f(x) = \frac{5-2x}{4}$, $g(x) = x^2+1$

find :

a) $fg(x) \rightarrow f(g(x)) = \frac{5-2x}{4} = \frac{5-2(x^2+1)}{4}$

$= \frac{5-2x^2-2}{4}$

$= \boxed{\frac{3-2x^2}{4}}$

b) $gf(x) \rightarrow g(f(x)) = x^2+1$

$= \left(\frac{5-2x}{4}\right)^2 + 1$

$= \frac{(5-2x)^2}{4^2} + 1$

$= \frac{25-20x+4x^2}{16} + \frac{1 \times 16}{1 \times 16}$

$= \frac{4x^2-20x+25+16}{16}$

$= \boxed{\frac{4x^2-20x+41}{16}}$

$f(x) = \frac{3x-1}{5}$, $g(x) = 3x+2$ $\qquad \frac{\overline{a}}{a} = \overline{a^2}$

find :

a) $gg(x) \rightarrow g(g(x)) = 3x+2 \rightarrow 3(3x+2)+2$

$= 9x+6+2$

$= \boxed{9x+8}$

b) $ff(x) \rightarrow f(f(x)) = \frac{3x-1}{5} \rightarrow \frac{3\left(\frac{3x-1}{5}\right)-1}{5}$

$= \frac{\frac{9x-3}{5} - \frac{1 \times 5}{1 \times 5}}{5}$

$= \frac{\frac{9x-3-5}{5}}{5}$

$= \frac{\frac{9x-8}{5}}{5} = \boxed{\frac{9x-8}{25}}$

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Sub:

Date:

we can get normal and substitute with no.

numbers composition: → we can start with inside

$f(x) = \frac{3x-1}{5}$, $g(x) = 3x+2$

find:

start with inside if its number

a) $fg(1) \longrightarrow f(g(1)) = 3(1)+2 = 5$

$\frac{3(5)-1}{5} = \frac{14}{5} = \boxed{28}$

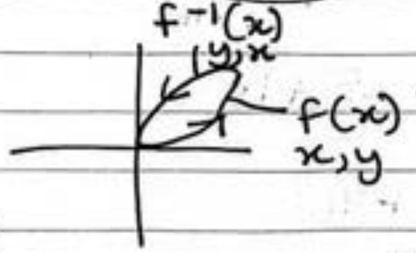
b) $gf(-2) \longrightarrow g(f(-2)) = \frac{3(-2)-1}{5} = \frac{-6-1}{5} = \frac{-7}{5} = -1.4$

$3(-1.4)+2 = \boxed{-2.2}$

inverse functions:

$f(x) \rightleftharpoons f^{-1}(x)$

$g(x) \longrightarrow g^{-1}(x)$



f(x)	$f^{-1}(x)$
2, 3	3, 2
0, 4	4, 0

$$x \longrightarrow \overset{f(x)}{\boxed{\times 2 \mid -5 \mid \div 3}} \longrightarrow \frac{2x-5}{3}$$ main

$$\frac{3x+5}{2} \longleftarrow \underset{f^{-1}(x)}{\boxed{+5 \mid \times 3 \mid \div 2}} \longleftarrow x$$

$f(x) = \boxed{}$

① change $f(x) = y$

② switch all x to be y and y to be x

③ make y subject again

$f(x) = \frac{3x-5}{2}$, find $f^{-1}(x)$?

$y = \frac{3x-5}{2} \longrightarrow x = \frac{3y-5}{2}$

$2x = 3y - 5$

$3y = 2x+5$

$f^{-1}(x)$

$\boxed{y^{-1} = \frac{2x+5}{3}}$

$f^{-1}(x) = \frac{2x+5}{3}$

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Sub:

Date:

$h(x) = \frac{3x-7}{5+2x}$, find $h^{-1}(x)$

$y = \frac{3x-7}{5+2x} \longrightarrow x = \frac{3y-7}{5+2y}$

$x(5+2y) = 3y-7$

$5x+2yx = 3y-7$

$5x+7 = 3y-2yx$

$5x+7 = (3-2x)y$

$\boxed{y^{-1} = \frac{5x+7}{3-2x}}$ g $h^{-1}(x) = \frac{5x+7}{3-2x}$

Scanned with CamScanner

Sub:

Date: 23/2/2024

Inverse of quadratic functions.

$f(x)y= ax^2 + bx + c$

$f(x) = ax^2 + bx$

find the inverse ?

① $f(x) = y$

② switch y with x and x with y, $x = ay^2+by+c$

$x - c = ay^2+by$ → completing the square.

example 1 :

$f(x) = x^2 - 8x + 6$; $x > 0$

find $f^{-1}(x)$?

Ans.

$y = x^2 - 8x + 6$

$x = y^2 - 8x + 6$

$x - 6 = y^2 - 8y$

$x - 6 = (y + \frac{-8}{2})^2 - (\frac{-8}{2})^2$

$x - 6 = (y - 4)^2 - (-4)^2$

$x - 6 = (y-4)^2 - 16$

$x - 6 + 16 = (y-4)^2$

$\sqrt{x+10} = \sqrt{(y-4)^2}$

$y - 4 = \sqrt{x+10}$

$y = 4 + \sqrt{x+10}$

$\boxed{f^{-1}(x) = 4 + \sqrt{x+10}}$

simple:

$(x+\frac{b}{2})^2 - (\frac{b}{2})^2 \overbrace{(+c)}^{\text{depends on question}}$

~~non-simple:~~

~~$(x+\frac{b}{2})$~~

Domain and range :

domain : possible x-values to be used in Function.

$x \geq 5.$

range : possible y-values to be used in function.

$f(x) = \frac{a}{b}$ $b \neq 0$

excluded values from domain :

rational function	square root function
$f(x) = \frac{1}{x}, x \neq 0$	$f(x) = \sqrt{x}, x \leq 0$
$f(x) = \frac{2x}{3-x}, x \neq 3$	$f(x) = \sqrt{x+3}, x+3 < 0$ $x < -3$
$g(x) = \frac{2}{5x-2}, x \neq \frac{2}{5}$	$f(x) = \sqrt{2x-5}, 2x-5 < 0$ $2x < 5$ $x < \frac{5}{2}$

*if you cant get, make equation with denominator and assume it equals 0.

$f(x)$	$f^{-1}(x)$
domain:- $x \geq 5$	range:- $y \geq 5$
range:- $y \geq 2$	domain:- $x \geq 2$
x, y	y, x

$f(x) = \frac{3-x}{4+x}, \boxed{x > -4} \rightarrow$ domain

a) find range of $f^{-1}(x)$?

$y > -4$

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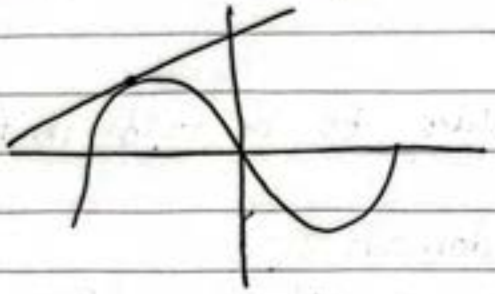
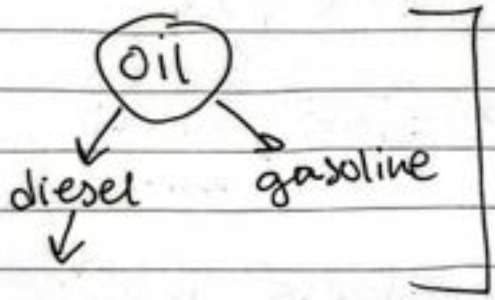
Sub:
Date: 25/2/2024

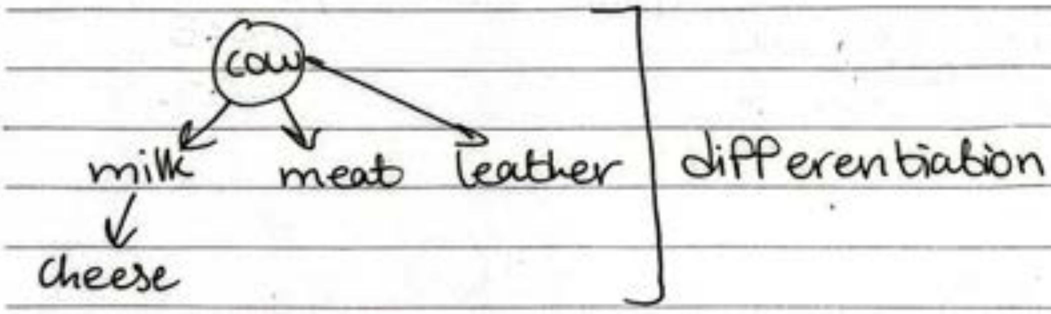
Calculus.

Differentiation.

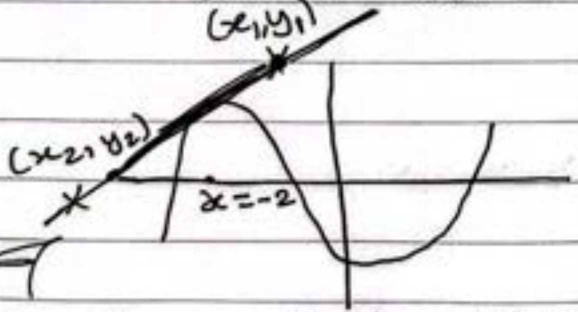
gradients of tangents to a curve







purpose: how to find gradients of a tangent to a curve.



Q - estimate gradient of the tangent to curve C at $x=-2$?

to get gradient without a graph.] main objective

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$\boxed{\frac{dy}{dx}}$ dy → difference of y

dx → difference of x

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* $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$

$\frac{dy}{dx}$ - (gradient function) or (first derivative)

example:

$y = ax^m + bx^m + cx^b$ $\frac{dy}{dx} = ??$

steps :-

1- multiply power of 'x' by coefficient of 'x'
2- decrease 1 from the power of 'x'.

$y = 3x^2 + 4x^5$
find $\frac{dy}{dx}$?
$3\overset{\curvearrowleft}{x}^2 + 4\overset{\curvearrowleft}{x}^5$
$= \boxed{6x + 20x^4}$

find $\frac{dy}{dx}$?

① $y = 3x^2 + 5x^4 - 3x^3 = 6x + 20x^3 - 9x^2 = \frac{dy}{dx}$

② $y = 12x^3 - 7x^2 + 5x^4 = 36x^2 - 14x + 20x^3 = \frac{dy}{dx}$

③ $y = x^4 - 2x^3 + 5x^6 + 3x = 4x^3 - 6x^2 + 30x^5 + 3 = \frac{dy}{dx}$

* if y has x' power 1, remove 'x' and add coefficient only.

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Sub:

Date:

find $\frac{dy}{dx}$?

① $y = 3x^2 - 17x \cancel{+24} - 5x^3 = 6x - 17 - 15x^2 = \frac{dy}{dx}$

② $y = 4x^3 + 14x - 2x^2 \cancel{+3} = 12x^2 + 14 - 4x = \frac{dy}{dx}$

* if function has a number only, we cant use as it becomes 0.

---

find $\frac{dy}{dx}$? *make sure there is no x in denominator.

① $y = 3x^2 + 4x - \frac{17}{\boxed{x}} = y = 3x^2 + 4x - 17x^{-1}$ $\Big|\ \frac{dy}{dx} = 6x + 4 + \frac{17}{x^2}$

$\frac{dy}{dx} = 6x + 4 - 17x^{-1}$ *optional

② $y = 12x - 4 + 3x^2 + \frac{12}{5\boxed{x}} = y = 12x - \cancel{4} + 3x^2 + \frac{12x^{-1}}{5}$

$\frac{dy}{dx} = 12 + 6x - \frac{12}{5}x^{-2}$

$\frac{dy}{dx} = 12 + 6x - \frac{12}{5x^2}$

---

find $\frac{dy}{dx}$?

① $y = 3x - 2x^2 + 5x^3 + \frac{2}{\boxed{x}} = 3x - 2x^2 + 5x^3 + 2x^{-1}$

$\frac{dy}{dx} = 3 - 4x + 15x^2 - 2x^{-2}$

$\frac{dy}{dx} = 3 - 4x + 15x^2 - \frac{2}{x^2}$

② $y = \cancel{14} + \frac{5}{3\boxed{x}} + 4x^2 = \cancel{14} + \frac{5x^{-1}}{3} + 4x^2$

$\frac{dy}{dx} = \frac{-5x^{-2}}{3} + 8x$

$\frac{dy}{dx} = \frac{-5}{3x^2} + 8x$

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Objectives:
- finding $\frac{dy}{dx}$
- gradient of the tangent
- equation of the tangent to a curve
- stationary point / turning point.

Applications:
- maximum and minimum volume or area
- kinematics.

---

to get gradient of tangent:

① get $\frac{dy}{dx}$

② replace the given 'x' value into $\frac{dy}{dx}$.

example:

curve C, $y = 3x^2 - 4x + 5$

find gradient of tangent at $x = 2$.

$\frac{dy}{dx} = 6x - 4$

$m = \frac{dy}{dx} = 6(2) - 4 = 12 - 4 = \boxed{8}$

---

curve C, $y = 3x + \frac{2}{x} + 2x^2$

find gradient at $x = -2$.

$y = 3x + 2x^{-1} + 2x^2$

$\frac{dy}{dx} = 3 - 2x^{-2} + 4x$

$m = \frac{dy}{dx} = 3 - \frac{2}{x^2} + 4x = 3 - \frac{2}{(-2)^2} + 4(-2) = \boxed{-5.5}$

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curve C, $y = kx^3 + 4x^2 - 5x$ has gradient 5 at $x = 2$, find k.

$$\frac{dy}{dx} = 3kx^2 + 8x - 5$$

$$m = \frac{dy}{dx} = 3kx^2 + 8x - 5 = 5 = 3k(+2)^2 + 8(+2) - 5$$

$$5 = 3k(4) + 16 - 5$$

$$5 \overset{\longleftarrow}{=} 12k + 11$$

$$5 - 11 = 12k$$

$$-6 \overset{\leftarrow}{=} 12k$$

$$k = \frac{-6}{12}$$

$$k = -0.5$$

---

equation of the tangent :

$$y = \underset{\substack{\downarrow \\ \text{gradient}}}{\textcircled{m}}x + c$$

---

example :

curve C, $y = 3x^3 - 7x^2 + 5$
find equation of tangent
at $P(-1, 4)$?

Ans- $9x^2 - 14x = \frac{dy}{dx}$

$$m = \frac{dy}{dx} = 9(-1)^2 - 14(-1) = 9 + 14 = \boxed{23}$$

$$y = mx + c$$

$$4 = 23(-1) + c$$

$$4 \overset{\leftarrow}{=} -23 + c$$

$$4 + 23 = c$$

$$27 = c$$

$$y = 23x + 27$$

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Sub:
Date:

Q- find equation of the tangent to graph of $y=18-3x-x^2$ at $x=4$. give ans as $y=mx+c$.

$y = 18-3(4)-(4)^2$

$\frac{dy}{dx} = -3-2x \quad y=-10$

$m = \frac{dy}{dx} = -3-2(4) = -11$

* if we dont have y, subtitvte into main equation to get y

$y=mx+c$, $(4,-10)$

$-10 = -11(4)+c$

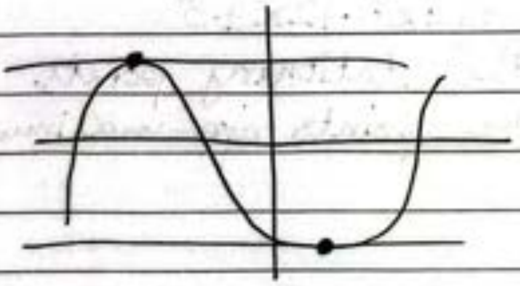
$-10 = -44+c$

$44-10=c$

$34=c$

$\boxed{y=-11x+34}$

Stationary points and its nature.



how to find stationary points?

① get $\frac{dy}{dx}$

② equate $\frac{dy}{dx}=0$

③ solve for x

④ replace x in curve equation to get y.

examples

curve C has equation $y=x^3-12x+5$.
find cordinates of stationary points.

$\frac{dy}{dx} = 3x^2-12$

$3x^2-12=0$

$\sqrt{3x^2}=\sqrt{12}$

$x = \pm 2$

at $x=2$, $y=(2)^3-12(2)+5$

$y=-11$

at $x=-2$, $y=(-2)^3-12(-2)+5$

$y=21$

$x_1=2 \mid x_2=-2$

Continue differentiation

Sub:
Date: 28/2/2024

✗ -How to determine weather ~~sta~~ stationary point is maximum or minimum.

during ramadan:
same time,
weekend lessons will
solve in pp.

after finding coordinates of stationary points;

a) find $\frac{d^2y}{dx^2}$ (differentiate again)

b) replace x values in $\frac{d^2y}{dx^2}$

- $\rightarrow$ if $\frac{d^2y}{dx^2}$ = + answer, the point is minimum.
- $\rightarrow$ if $\frac{d^2y}{dx^2}$ = - answer, the point is maximum.

---

example:

curve C has equation $y=x^3-12x+5$

a) find coordinates of the stationary points.

b) determine weather the points are maximum or minimum.

Ans a) $\frac{dy}{dx}=3x^2-12$

$0=3x^2-12$

$12=3x^2$

$\frac{12}{3}=x^2$

$\sqrt{4}=\sqrt{x^2} \longrightarrow x=\textcircled{\pm}2$

important ✗

$y=2^3-12(2)+5$

$y=-11$

$y=-2^3-12(-2)+5$

$y=21$

$(2,-11)\ (-2,21)$

Ans b) $\frac{d^2y}{dx^3}=6x$

✗ at $x=2$ $(2,-11)$

$\frac{d^2y}{dx^2}=6(2)=\boxed{12}$

at $x=-2$ $(-2,21)$

$\frac{d^2y}{dx^2}=6(-2)=\boxed{-12}$

minimum $=(2,-11)$ & maximum $=(-2,21)$

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Applications:

① maximum / minimum volume or area / variable

a) find an expression for volume, area or variable in terms of x. ↘ equation of the function

b) find $\frac{dv}{dx} \Big| \frac{dA}{dx} \Big/ \frac{dy}{dx}$

c) equate $\frac{dv}{dx} \Big/ \frac{dA}{dx} \Big/ \frac{dy}{dx} = 0$

d) solve for x

e) substitute x in volume, area or variable.

② Kinematics

$$v=\frac{ds}{dt} \qquad a=\frac{dv}{dt}$$

$$\underset{s}{x} \longrightarrow v \longrightarrow a$$

displacement velocity acceleration

$$s=vt \longrightarrow v=\frac{s}{t}$$

→ particle is stationary, $v=0$

→ particle is at rest, $V=0$

→ instantaneously, $V=0$

→ particle changes direction at point A, $V=0$

velocity $=\frac{ds}{dt}$, acceleration $=\frac{dv}{dt}$

Sub:

Date: 1/3/2024

Continue Differentiation

Kinematics:

ex:

$x = 3t^2 - 18t + 45$

the particle moves from A to B. it stops at B find displacement A to B?

$V = \frac{dx}{dt} = 6t - 18$ $\boxed{V,=0}$

$0 = 6t - 18$

$18 = 6t$

$\frac{18}{6} = t$

$\underline{3 = t}$

$3(3)^2 - 18(3) + 45$

$\underline{x = 18 \text{ metres.}}$

---

maximum or minium speed :

① $v = \frac{ds}{dt}$

② $a = \frac{dv}{dt}$

③ equate $\frac{dv}{dt} = 0$ $(a = 0)$

④ solve for t

⑤ replace t in $\frac{ds}{dt}$ (v)

Sub:

Date:

Algebraic Proof:

→ whole numbers or integer: $n, n+1, n+2, n+3$ ——

$n-3, n-2, n-1, n$ ——

→ odd numbers: $2n+1, 2n+3, 2n+5, 2n+7$ ——

$2n-3, 2n-1, 2n+1, 2n+3$ ——

→ even numbers: $2n, \overset{2n+2}{\wedge}2n+4, 2n+6, 2n+8$ ——

$2n-4, 2n-2, 2n, 2n+2$ ——

→ multiples: of 2 or $2[\_\_\_]$

$4n^2+6n-8 = 2(2n^2+3n-4)$

of 3 or $3[\_\_\_]$

$3n^2+9n-15 = 3(n^2+3n-5)$

consecutive numbers: كل التوالي

2 consecutive odd

$2n-1, 2n+1, 2n+3$

square of $(\ )^2$

cube of $(\ )^3$

→ add, plus, sum, total (+)

→ minus, subtract, decrease (-)

→ product, times (X)

→ quotient (÷)

→ one more than a multiple of 4.

$4n^2-24n+5$

$4+1$

$4n^2-24n+4+1$

$4.(n^2-6n+1)+1$

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Sub:
Date: 3/3/2024

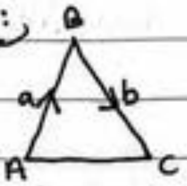
Vectors

- Missing vector.
- Given ratio in vectors.
- Parallel vectors.
- Points on a straight line.
- Column vectors.
- Vectors in polygons.
- similarity in vectors.
- Vector geometry.

Quantity

Scalar	Vectors
↓	↓
magnitude only	magnitude & direction
↓	↓
time	velocity
temp.	displacement
age	
speed	
distance	

missing:

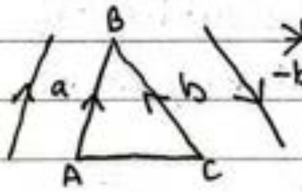


find $\overrightarrow{AC}$ in terms of a & b?

starting point ending point

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = a + b$



*to go opposite direction, take negative

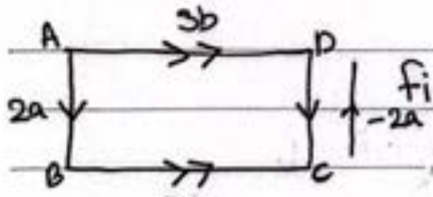
find $\overrightarrow{AC}$ in terms of a & b?

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = a - b$

examples

ABCD is a rectangle



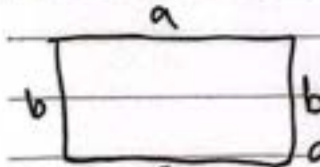
find $\overrightarrow{AC}$ and $\overrightarrow{BD}$ in terms of a and b?

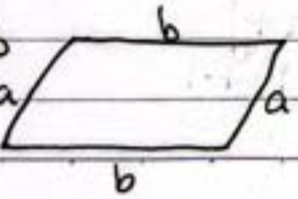
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

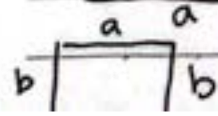
$\overrightarrow{AC} = 2a + 3b$

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$

$\overrightarrow{BD} = 3b - 2a$







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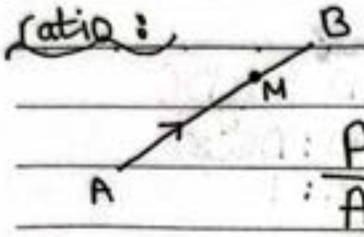
ratio :



$AM : MB = 3 : 1$

$\overrightarrow{AB} = 2a + b$

find AM and MB in terms of a & b?

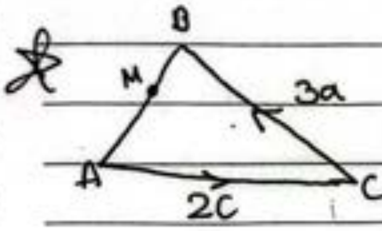
$\frac{2a+b}{4}$

$= \frac{1}{2}a + \frac{1}{4}b$

$\boxed{\overrightarrow{MB} = \frac{1}{2}a + \frac{1}{4}b}$

$\overrightarrow{AM} = 3(\frac{1}{2}a + \frac{1}{4}b)$

$\boxed{\overrightarrow{AM} = \frac{3}{2}a + \frac{3}{4}b}$



$AM : MB = 3 : 2$

find $\overrightarrow{AM}$ and $\overrightarrow{MB}$?

*geb ditance first by what we have.

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\boxed{\overrightarrow{AB} = 2c + 3a}$

$\overrightarrow{AM} = 3(\frac{2}{5}c + \frac{3}{5}a)$

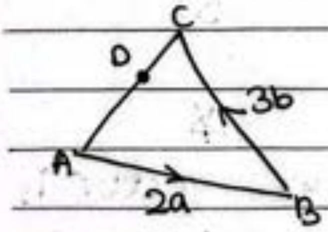
$\boxed{\overrightarrow{AM} = \frac{6}{5}c + \frac{9}{5}a}$

$\overrightarrow{MB} = 2(\frac{2}{5}c + \frac{3}{5}a)$

$\boxed{\overrightarrow{MB} = \frac{4}{5}c + \frac{6}{5}a}$

$\frac{2c+3a}{5}$ — $3+2$

$= \frac{2}{5}c + \frac{3}{5}a$ *



$\overrightarrow{AB} = 2a$, $\overrightarrow{BC} = 3b$, $AD : DC = 2 : 1$

find $\overrightarrow{BD}$?

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\boxed{\overrightarrow{AC} = 2a + 3b}$

$\frac{2a+3b}{3} = \boxed{\frac{2}{3}a + b}$

$\overrightarrow{AD} = 2(\frac{2}{3}a + b)$

$\boxed{\overrightarrow{AD} = \frac{4}{3}a + 2b}$

$\boxed{\overrightarrow{DC} = \frac{2}{3}a + b}$

$\overrightarrow{BD} = 3b + (-(\frac{2}{3}a + b))$

$\overrightarrow{BD} = 3b - \frac{2}{3}a - b$

$\boxed{\overrightarrow{BD} = 2b - \frac{2}{3}a}$

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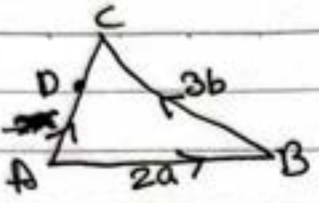


$\overrightarrow{AB} = 2a$, $\overrightarrow{BC} = 3b$, $\overrightarrow{AD} = 2\overrightarrow{DC}$

find $\overrightarrow{BD}$?

$AD : DC$
$2 : 1$

$\underline{\overrightarrow{AC} = 2a + Bb}$

$\overrightarrow{AD} = 2(2/3a + b)$

$\underline{\overrightarrow{AD} = 4/3a + 2b}$

$\frac{2a+3b}{3}$

$\overrightarrow{DC} = 2/3a + b$

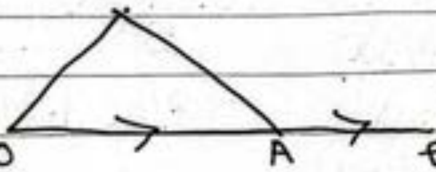
$\overrightarrow{BD} = 3b + (-(2/3a + b))$

$\overrightarrow{BD} = 3b - 2/3a - b$

$\boxed{\overrightarrow{BD} = 2b - 2/3a}$

---

hint:



$OA : AE = 5 : 3$

$\overrightarrow{OA} = a$

*we have one of ratios given.

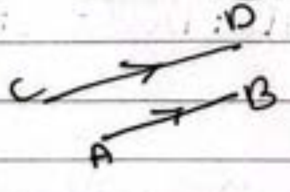
$OA : AE$
$a : x$
$5 : 3$

$x = \frac{3 \times a}{5}$

$x = \frac{3a}{5}$

---

parallel vectors:



multiple

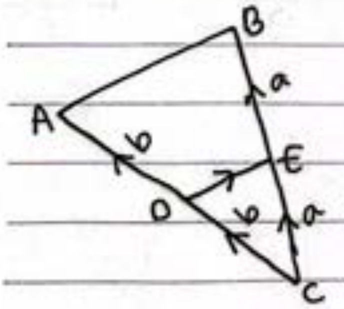
$\overrightarrow{AB} = k\,\overrightarrow{CD}$

$\overrightarrow{CD} = k\,\overrightarrow{AB}$

*when vector is a multiple of other:

1 - get 2 vectors.

2 - get a ratio between them



$\overrightarrow{CE} = a$

$\overrightarrow{CD} = b$

E is midpoint of $\overrightarrow{CB}$

D is midpoint of $\overrightarrow{CA}$

show that:

$\overrightarrow{DE}$ is parallel to $\overrightarrow{AB}$?

↳ 2n.

Ans: $\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE}$

$\boxed{\overrightarrow{DE} = a - b}$

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\overrightarrow{AB} = -2b + 2a$

$\boxed{\overrightarrow{AB} = 2a - 2b}$

$2(a-b) = \overrightarrow{AB} \div \overrightarrow{DE}$

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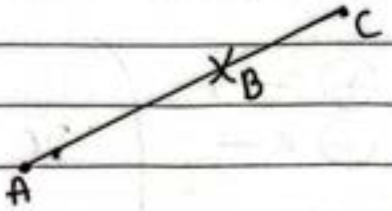
Points on a straight line :

- 3 points on a straight line.
- Prove that M, N and E represent a straight line.



steps :

① choose :

- AB to BC	ratio
or - AB to AC	
or - BC to AC	

② get a ratio between them

③ mention that there is a common point.

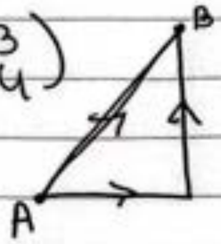
Column vector

Sub:
Date: 6/3/2024

$\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ → horizontal movment
→ vertical movment

A → starting point
B → ending point

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$



x is ⊕ →
x is ⊖ ←

y is ⊕ ↑
y is ⊖ ↓

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

---

the magnitude of the vector (length)
*use pythagoras

$\underbrace{|\overrightarrow{AB}|}_{\text{magnitude}} = \sqrt{x^2+y^2}$

---

operations:

$a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$, $c = \begin{pmatrix} -2 \\ -7 \end{pmatrix}$

find:

① $a+b = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

② $3a = 3\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ *multiply by all

③ $2a+3b = 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3\begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -15 \\ 12 \end{pmatrix} = \begin{pmatrix} -11 \\ 18 \end{pmatrix}$

④ $|a+2b-c| = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -5 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -10 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 18 \end{pmatrix}$

$\sqrt{(-6)^2+18^2} = \boxed{6\sqrt{10}}$

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* hint:

① $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, find $\overrightarrow{BA}$

$$\overrightarrow{BA} = -\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$
$$\overrightarrow{BA} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

---

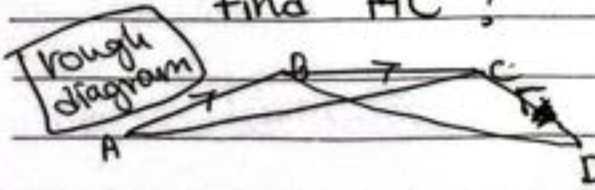
② parallel

$$\overrightarrow{AB} = K\,\overrightarrow{CD}$$
$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$
$$\overrightarrow{CD} = \begin{pmatrix} 18 \\ 12 \end{pmatrix}$$
$$\overrightarrow{CD} = 6\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$
$$\overrightarrow{CD} = 6\overrightarrow{AB}$$

---

③ $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, $\overrightarrow{DC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

find $\overrightarrow{AC}$?



$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$\overrightarrow{AC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$
$$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

find $\overrightarrow{BD}$?

$$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$$
$$\overrightarrow{BD} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$
$$\overrightarrow{BD} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$
$$\overrightarrow{BD} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

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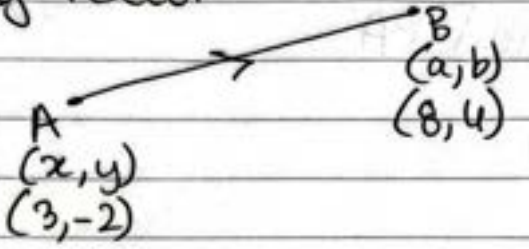
point and vector:

① finding vector



~~B~~

Vector = ending point − starting point

find $\overrightarrow{AB}$?

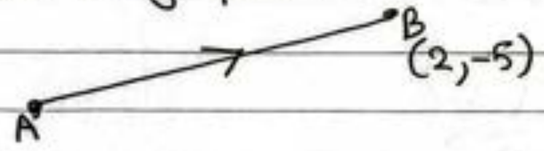
$\overrightarrow{AB} = B - A$

$\overrightarrow{AB} = \begin{pmatrix}8\\4\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix}5\\6\end{pmatrix}$

---

② Finding starting point



$\overrightarrow{AB} = \begin{pmatrix}5\\3\end{pmatrix}$, Find coordinates of A ?

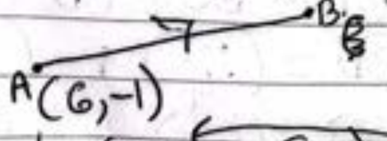
$\begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}2\\-5\end{pmatrix} - \begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix}_{(A)} = \begin{pmatrix}2\\-5\end{pmatrix} - \begin{pmatrix}5\\3\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-3\\-8\end{pmatrix}$ $\underline{A(-3,-8)}$

---

③ finding ending point



$\overrightarrow{AB} = \begin{pmatrix}5\\6\end{pmatrix}$ | $\begin{pmatrix}5\\6\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}6\\-1\end{pmatrix}$

$\begin{pmatrix}5\\6\end{pmatrix} + \begin{pmatrix}6\\-1\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}11\end{pmatrix} = \begin{pmatrix}x\end{pmatrix}$ B(11,5)

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Date: 8/3/2

Sets and Venn diagram

set : a collection of items that have a common property.

naming sets :-
-upper case letters : A, B, X, Y, T

expressing the set :

① listing method
$X = \{2, 4, 6, 8, 10\}$
② descriptive method
$X = \{x : x \text{ is an even number}, 2 \leq x \leq 10\}$

( ) = parenthesis
[ ] = bracket
{ } = braces

---

* list element of each set :

a) $x = \{x : x \text{ is odd no.}, 1 < x \leq 13\}$
$x = \{3, 5, 7, 9, 11, 13\}$

b) $T = \{t : t \text{ is a multiple of } 3, t < 10\}$
$T = \{3, 6, 9\}$

---

* symbols :
$\in, \notin, \subset, \not\subset$

$\in$ = belongs to / member of.

$\notin$ = not member of / doesnt belong.

$\subset$ = part of / subset of.

$\not\subset$ = not part of / not subset of.

Sub:
Date:

example : ✓✓✓

$x = \{2, ③, 4, 5, 8, ⑪, 17, 18, 23\}$

put appropriate symbol $(\in, \notin, \subset, \not\subset)$

① $3 \ \underline{\in} \ x$

② $\{2, 4\} \ \underline{\subset} \ x$

③ $\{7\} \ \underline{\not\subset} \ x$

④ $9 \ \underline{\notin} \ x$

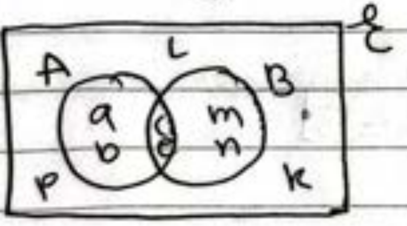
⑤ $11 \ \underline{\in} \ x$

⑥ $\{2, 3, 4, 31\} \ \underline{\not\subset} \ x$

* without braces, either $\in$ or $\notin$

* set inside braces is all ~~[illegible]~~ avaliable.

Venn Diagram :



$A + A' = ℰ$

$B + B' = ℰ$

(drop, flip, put)

demorgan's law

$A' \cup B' = (A \cap B)'$

$A' \cap B' = \underbrace{(A \cup B)'}_{\text{cancel union}}$

cancel intersection

intersection

* ℰ = universal set, $\cap$ = common

$\cup$ = union (all elements), A' = prime, compliment (anything that isnt A)

find:

① ℰ = a, b, c, e, m, n, p, k, l

② A = a, b, c, e

③ B = c, e, m, n

④ $A \cap B$ = c, e

⑤ $A \cup B$ = a, b, c, e, m, n

⑥ A' = l, p; k, m, n

⑦ B' = a, b, l, p, k

⑧ $A' \cap B$ = m, n

⑨ $A \cup B'$ = a, b, c; e, l, p, k

⑩ $A' \cup B'$ = ~~[illegible]~~ A' = l, m, n, p, k, B' = a, b, l, p, k

⑪ $A' \cap B'$ =

$A' \cup B'$ = l, m, n, a, b, p, k

$= \underbrace{(A \cup B)'}_{\text{cancel all}}$ = l, p, k

or $\underbrace{(A \cap B)'}_{\text{cancel intersection}}$ = l, p, k, a, b, m, n

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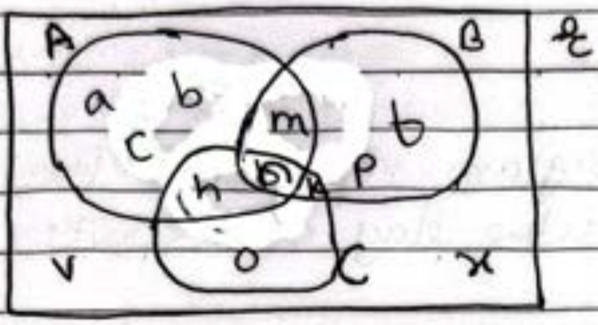


*if brackets, do them first.

Finds:

① $A \cup B \cup C = a, b, c, m, n, p, b, h, k, o$

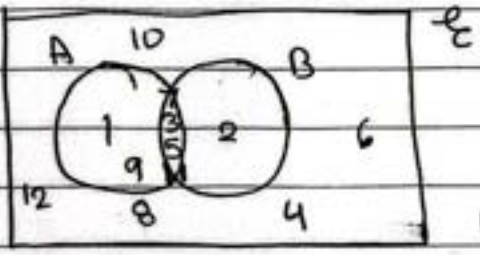
② $A \cap B \cap C = n$

③ $A \cap (B \cup C) = A \cap (m, n, k, p, b, h, o) = \underline{h, m, n}$

④ $A' \cap (B \cap C) = A^c \cap (n, k) = (t, p, k, o) \cap (n, k) = \underline{k}$
v,x

⑤ $(A \cup B) \cap (B \cup C) = (a, b, c, m, n, h, k, t, p) \cap (m, n, k, t, p, h, o)$
$= \underline{m, n, k, \cancel{k}, b, p, h}$

number of elements:



*if 'n' comes before, we should count.

1,2,3,4,5,6,7,8,9,10,11,12

ℰ = {x : x is an integer, $0 < x \leq 12$}

A = {a : a is an odd number}
1,3,5,7,9,11

B = {b : b is a prime number}
2,3,5,7,11

(i) fill venn diagram.

(ii) ① $n(A \cap B) = 4$

② $n(A' \cup B) = n((2,4,6,8,10,12) \cup (2,3,7,5,11))$
$= 10$

③ $A \cup B = 7, 1, 9, 5, 3, 11, 2$

④ $n(A' \cup B') = n(A \cap B)' = 8$

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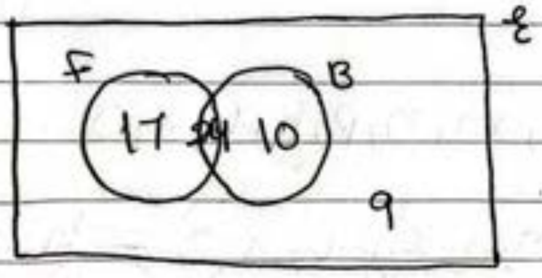
example:

Q- the following venn diagram represents the number of students who play some sports.



find:

(i) $n(F) = 17 + 24 = 41$

(ii) $n(F \cup B) = 17 + 24 + 10 = 51$

(iii) $n(F \cup B') = 17 + 24 + 9 = 50$

↓ 17,24 ↓ 17,9

(iv) $= n(F' \cup B') = n(F \cap B)'$
$= 9$

* if no intersection and question asks for that $F \cap B = \emptyset$ (fai)

$n(F \cap B) = 0$ ↓ not possible

↓ no intersection

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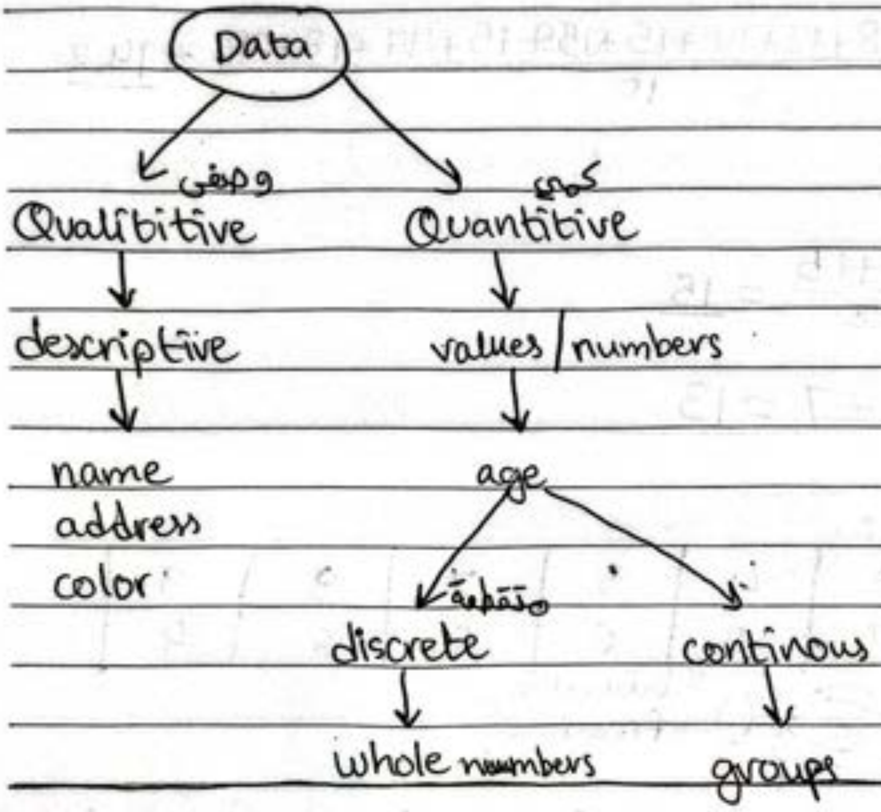
Date: 9/3/2024

Statistics 1

- averages and range
- quartiles and comparison between data
- cumilative frequency
- histogram



Averagess mean, median, mode

$$\text{mean} = \frac{\text{sum of all data values}}{\text{no. of values}}$$

mode = most repeated

median = middle value of an orderd data.

$\longrightarrow$ 2 values $\longrightarrow \frac{\text{add them both}}{2}$

range = higest value - lowest value

excercise

here are the scores of 10 students in a quiz

13, 7, 8, 15, 17, 15, 20, 18, 14, 15

find: 7, 8, 13, 14, 15, 15, 15, 17, 18, 20

a) the mean

$$\frac{7+8+13+14+15+15+15+17+18+20}{10} = \underline{14.2}$$

b) the mode

$\underline{15}$

c) the median

$$\frac{15+15}{2} = \underline{15}$$

d) the range

$$20 - 7 = \underline{13}$$

---

frequency table:

scores	3	4	5	7	8	9
frequency	7	2	6	5	6	4

data value

sum of total ← $\dfrac{\sum x f}{\sum f}$ → frequency

find:

a) the mean $\dfrac{(3\times7)+(4\times2)+(5\times6)+(7\times5)+(8\times6)+(9\times4)}{7+2+6+5+6+4} = \underline{5.9\dot{3}}$

b) the modal → highest frequency.

$\underline{3}$

c) the median

if even, we take 2 ← $\dfrac{30}{2} = 15^{th}$ & 16^{th} $= \dfrac{5+7}{2} = \dfrac{12}{2} = 6$

d) the range

$$9 - 3 = \underline{6}$$

Continous data :

	2.5 ↑	6 ↑	8.5 ↑
scores	$0<x\le5$	$5<x\le7$	$7<x\le10$
frequency	4	12	14

$a < x \le b$

midpoint $= \frac{a+b}{2}$

find:

$$\text{a) mean} = \frac{\Sigma \overset{\text{midpoint}}{\overset{\uparrow}{x}} f}{\Sigma f} = \frac{(2.5\times4)+(6\times12)+(8.5\times14)}{4+12+14} = \underline{6.7}$$

b) modal class → class with highest frequency

$= 7<x\le10$

combined mean
combined range

$$\text{(discrete data) mean} = \frac{\boxed{\Sigma x} \xrightarrow{} \text{sum of data}}{n \rightarrow \text{their number}}$$

$$\underset{\text{sum of data}}{\Sigma x} = \text{mean} \times n \rightarrow \text{number of data}$$

$$\text{* combined mean} = \frac{\Sigma x + \Sigma y}{n_1 + n_2}$$

$\Sigma x = n \times \text{mean}$
$\Sigma y = n \times \text{mean}$

set A	set B
$\Sigma x = 920$	$\Sigma y = 960$
$n = 30$	$n = 27$

$$\text{Overall mean} = \frac{\Sigma x + \Sigma y}{n_1 + n_2} = \frac{920+960}{30+27} = 32.98$$

$$\text{* combined range} = \underset{\text{value}}{\text{highest}} - \underset{\text{value}}{\text{lowest}}$$

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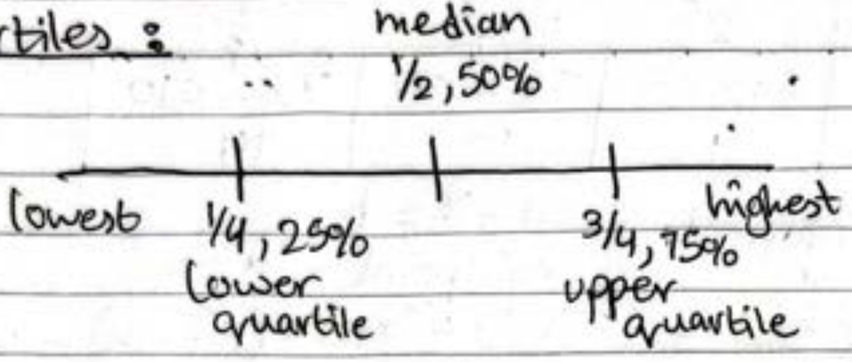
Date:

Quartiles :



*getting quartiles depend on weather 'n' is odd or even.

range = highest - lowest

interquartile range = upper quartile - lower quartile

even
↓
arrange → split into 2 groups equally → median of each group
→ first group is lower and second is upper.

example:
Q-scores for 10 students were as follows
7, 13, 6, 19, 15, 18, 14, 12, 20, 18
find inter quartile ranges
6, 7, 12, 13, 14, 15, 18, 18, 19, 20,
l.q. = 12 , u.q. = 18 , i.q. = 18 - 12 = $\boxed{6}$
Q-scores for 11 students were as follows
7, 13, 6, 19, 15, 18, 14, 12, 20, 18, 17
find inter quartile range:
6, 7, 12, 13, 14, ~~15~~, 17, 18, 18, 19, 20,

lower = 12 upper = 18
inter = 18 - 12 = $\boxed{6}$

odd

~~Et~~ arrange → cancel middle → arrange → get medians and subtract

Statistics 2

Sub:
Date: 10/8/2024

example:

the ages of 14 people were as follows:

12, 34, 56, 25, 33, 72, 16, 24, 50, 70, 66, 53, 13, 35

find interqurtile range

$$\underbrace{12, 13, 16, \underbrace{24}_{lq}, 25, 33, 34}, \underbrace{35, 50, 53, \underbrace{56}_{hq}, 66, 70, 72}$$

$$\text{interquartile} = 56 - 24 = 32$$

the ages of 11 people were as follows

11, 19, 20, 14, 8, 16, 9, 11, 17, 18, 11

find interquartile range

$$\underbrace{8, 9, \underbrace{\textcircled{11}}_{lq}, 11, 11}, \overset{\times}{14}, \underbrace{16, 17, \underbrace{\textcircled{18}}_{uq}, 19, 20}$$

$$\text{interquartile} = 18 - 11 = 7$$

$$\boxed{u.q. = \frac{3n}{4}, \quad l.q. = \frac{n}{4}} \ *$$

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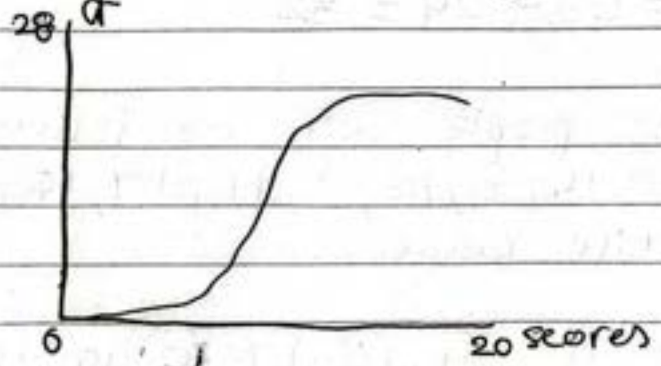
Cumilative frequency:

continous data:

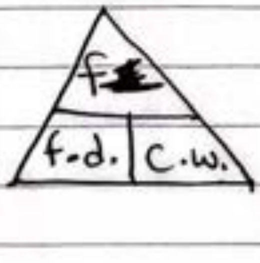
scores	$0<x\leq5$	$5<x\leq8$	$8<x\leq15$	$15<x\leq20$
frequency	3 (3)	7 (10)	8 (18)	7 (28)

draw cumilative frequency diagram



* to find median, half cumilative frequency then find the equivilant on x axis.

* frequency density = $\frac{\text{frequency}}{\text{Class width}}$



Class width: difference between first and last number $a<x\leq b$; $b-a=$

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Sub:

Date: 13/3/2024

Continue stabistics

Histogram → its a bar chart for continous data.

example:

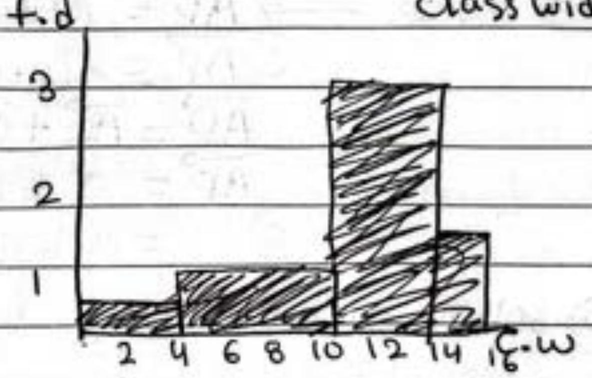
here is the score of 20 students in a quiz.

score	$0 < x \le 4$	$4 < x \le 10$	$10 < x \le 13$	$13 < x \le 15$
frequency	2	6	9	3

- draw histogram.

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

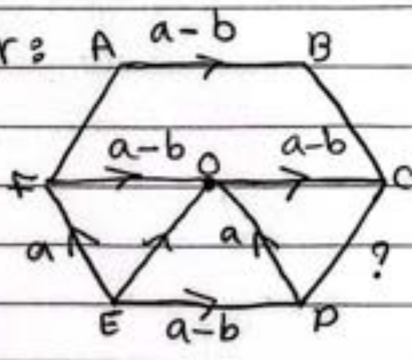
C.W.	F.d.
$4-0=\underline{4}$	$\frac{2}{4}=\underline{0.5}$
$10-4=\underline{6}$	$\frac{6}{6}=\underline{1}$
$13-10=\underline{3}$	$\frac{9}{3}=\underline{3}$
$15-13=\underline{2}$	$\frac{3}{2}=\underline{1.5}$



[illegible]

Vectors in polygon.

regular:

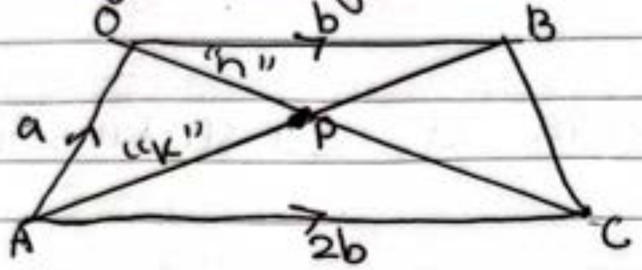


$\overrightarrow{DC} = ?$

$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$

$= a + a - b = \underline{2a - b}$

Vector geometry:



$\overrightarrow{AP} : \overrightarrow{PB}$?

steps:

① gets the diagonals → $\overrightarrow{OC}$ and $\overrightarrow{AB}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= -a + 2b$

$\underline{= 2b - a}$

② get the first vector by 2 ways → $\overrightarrow{AP} = k\overrightarrow{AB}$

① $\overrightarrow{AP} \underline{= k(a+b)}$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ $\underline{= a + b}$

$\overrightarrow{AP} = a + n\overrightarrow{OC}$

② $= a + n(2b-a)$

③ solve simultaneuos to get k.

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Sub: 3
Date: 15/[illegible]/2024

Transformations of functions

Translation (moving)	Reflection	Stretching

Translation (moving):

a) horizontal (x – axis)

↳ translation by $\binom{-a}{0}$

$y = f(x) \longrightarrow y = f(x+\overset{-a}{a})$

$y = f(x) \longrightarrow y = F(x-a)$

* add $\pm a$ to the x coordinates

b) vertical (y-axis)

↳ translation by $\binom{0}{a}$

$y = f(x) \longrightarrow y = f(x) + a$

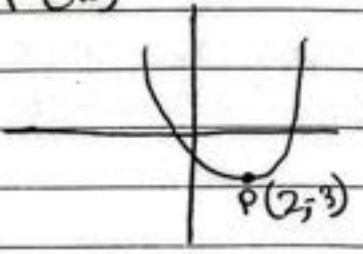
$y = f(x) \longrightarrow y = F(x) - a$

* add $\pm a$ to the y coordinates

example.

→ $y = f(x)$

a) tranlation by $\binom{-a}{0}$
b) translation by $\binom{0}{a}$
to describe



Q- find p on $y = f(x) + 2$

$P(2, -3+2)$

$\underline{P(2, -1)}$

Q - find p on $y = f(x+2)$

$y = f(x) - 2$

$P(2-2, -3)$

$\underline{P(0, -3)}$

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Sub:
Date:

* Describe Translation ~~Transformation~~:

a) i) $y=f(x) \longrightarrow y=f(x+a)$ $-a$

ii) $y=f(x) \longrightarrow y=f(x-a)$ $+a$

i) translation by $\begin{pmatrix}-a\\0\end{pmatrix}$

ii) translation by $\begin{pmatrix}a\\0\end{pmatrix}$

b) i) $y=f(x) \longrightarrow y=f(x)+a$

ii) $y=f(x) \longrightarrow y=f(x)-a$

i) translation by $\begin{pmatrix}0\\a\end{pmatrix}$

ii) translation by $\begin{pmatrix}0\\-a\end{pmatrix}$

c) $y=f(x) \longrightarrow y=f(\overset{-a}{x+a})+b$

translation by $\begin{pmatrix}-a\\b\end{pmatrix}$

Reflection:

a) $y=f(x) \longrightarrow y=f(-x)$

multiply (-1) by x-coordinates

$\boxed{\text{or}}$ change sign of x-coordinates.

b) $y=f(x) \longrightarrow y=-f(x)$

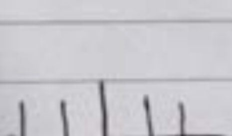
multiply (-1) by y-coordinates

$\boxed{\text{or}}$ change sign of y-coordinates

* Describe Reflection:

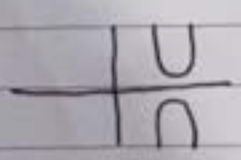
i) $y=f(x) \longrightarrow y=f(-x)$

reflection about y-axis



ii) $y=f(x) \longrightarrow y=-f(x)$

reflection about x-axis



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(changes in function's size)

Sub:
Date:

Stretching:

a) horizontal (x-axis)

$$y = f(x) \longrightarrow y = f(\overset{\rightarrow 1/a}{\textcircled{a}}x)$$

multiply $\frac{1}{a}$ by x-coordinates

b) vertical (y-axis)

$$y = f(x) \longrightarrow y = af(x)$$

multiply a by y-coordinates

✻ Describe stretching:

a) $y = f(x) \longrightarrow y = f(ax)$

stretching in the x direction by scale factor $\frac{1}{a}$.

b) $y = f(x) \longrightarrow y = af(x)$

stretching in the y direction by scale factor a.

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Sub:

Continue transformations of functions

Date: 16/3/2024

example 1:

curve S : $f(x) = \underline{(x)}^2 + 2\underline{(x)} + 5$

curve T : $g(x) = \underline{(x-3)}^2 + 2\underline{(x-3)} + 5$

describe the transformation that maps S onto T?

Ans - translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

example 2:

curve C : $f(x) = \underline{x}^2 - 7\underline{x} + \underline{2}$

curve T : $g(x) = \underline{x}^2 - 7\underline{x} + \underline{5}$

Ans - translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

example 3:

curve T : $f(x) = \underline{x}^2 - 6\underline{x} + 7$

curve T is transformed to S by translation of $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

find equation of S in simplest form.

Ans - curve S $= (x-2)^2 - 6(x-2) + 4$

$f(x) =$

example 4:

curve C : $f(x) = x^2 - 5x + \underline{4}$

C is transformed to S

a) find S : $y = f(x-4)$

Ans - S : $y = (x-4)^2 - 5(x-4) + 4$

b) find S : $y = f(x) - \underline{2}$

Ans - S : $y = x^2 - 5x + \overset{(4-2)}{2}$

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example 5 : curve C: $f(x) = x^2$

curve T : $g(x) = 3x^2 - 18x + 7$

by writing $g(x)$ in the form $a(x+b)^2+c$

describe the transformation that maps C onto T.

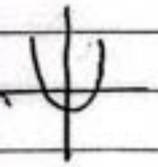
Ans –

$$\underbrace{3x^2 - 18x}\,+7$$
$$3(x^2 - 6x) + 7$$
$$3\left[\left(x + \frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2\right] + 7$$
$$\textcircled{3}\left((x-3)^2 - 9\right) + 7$$
$$3(x-3)^2 - 27 + 7$$
$$\boxed{3(x-3)^2 - 20}$$

stretching in 'y' by scale factor 'a'

stretching in y direction by scale factor 3, translation by $\begin{pmatrix} 3 \\ -20 \end{pmatrix}$

$$* \; f(x) = x^2 \longrightarrow g(x) = \textcircled{a}(x + \textcircled{b})^2 + \textcircled{c} \quad \begin{pmatrix} -b \\ c \end{pmatrix}$$

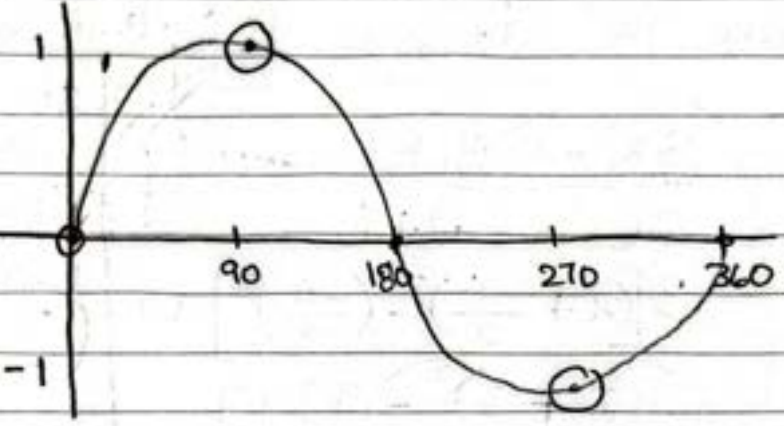


→ stretching in y-direction by scale factor a, translation by $\begin{pmatrix} -b \\ c \end{pmatrix}$.

Sub:
Date:

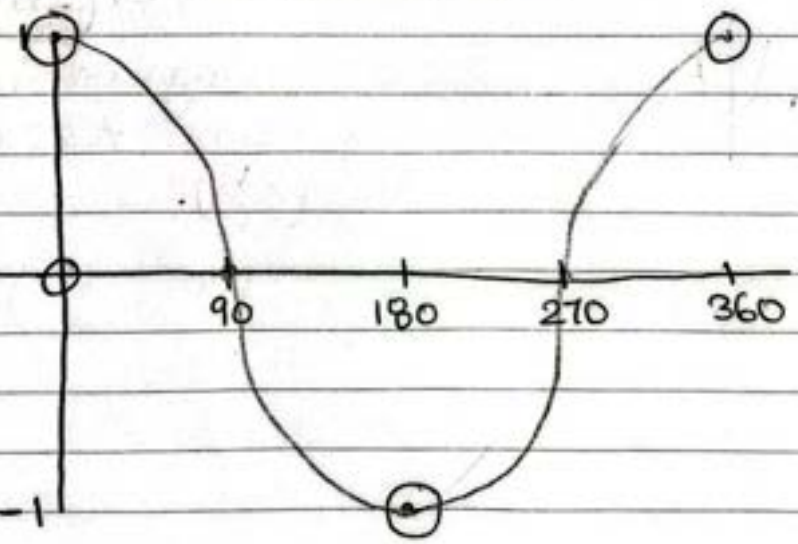
Trignometric graphs + its transformations:

$y = \sin x \ , \ 0 \leq x \leq 360$



(0,0)
(90,1) → max
(180,0)
(270,-1) → min
(360,0)

$y = \cos x \ , \ 0 \leq x \leq 360$



(0, 1) → max
(90, 0)
(180,-1) → min
(270, 0)
(360, 1) → max

* to get order pairs on calculator all at once:

→ mode
→ table
→ sin/cos/t
→ X

→ write start and end as 0 & 360
→ write 90 as step

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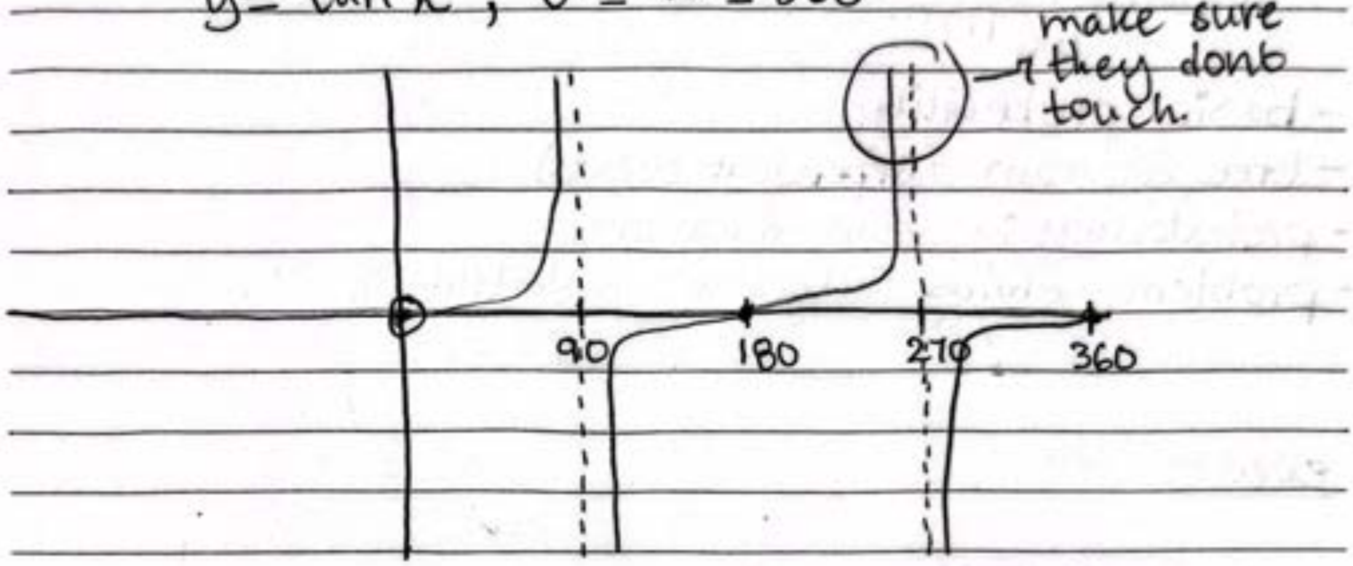
$y = \tan x \,, \quad 0 \leq x \leq 360$



$\underline{y = \sin x}$ & $\underline{y = \cos x}$

$y = \overset{\text{stretch}}{\textcircled{a}} \sin x / \cos x$

stretch of y
↑
$a = \frac{1}{2}$ [maximum value − minimum value]

$y = \textcircled{a} \underset{\cos x}{\sin x} + \textcircled{b}$

stretch — translating in y

$y = \textcircled{a} \underset{\cos}{\sin} (x + \textcircled{b})$

stretch — −b added to x

$c = \frac{1}{2}$ [maximum value − minimum value]
↓
~~num~~ translation of y

$y = \textcircled{a} \underset{\cos}{\sin} (x - \textcircled{b}) + \textcircled{c}$

stretch in y — +b added to x — added to y

$y = \underset{\cos}{\sin} (kx)$

k = number of maximum points

$k = \dfrac{360}{\text{difference between 2 maximums or 2 minimum points}}$

Sub:
Date: 18/4/2024

Probability

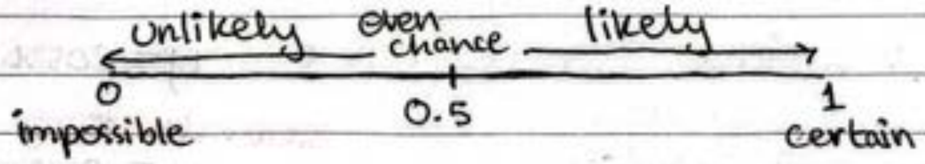
↳ the possibility that an event might happen or doesnt happen.

- basic probability
- tree diagram (dependent events)
- probability in venn diagram
- problem solving (algebraic probability).

---

events

probability scale



* $$\text{probability} = \frac{\text{possible outcomes}}{\text{total outcomes}}$$

example:

parallel

Find the probability of:

a) picking the letter L

$$\frac{3}{8}$$

b) picking the letter A

$$\frac{2}{8} = \frac{1}{4}$$

c) picking the letter that isnt P

$$\frac{7}{8}$$

Q- Katy has 5 white كورة , x red كورة and $(2x+1)$ yellow كورة. she picks a كورة at random. the probability that it is white is $\frac{1}{12}$ find the probability that it is yellow.

$$P(w) = \frac{5}{5+x+2x+1} = \frac{5}{3x+6}$$

$$\frac{5}{3x+6} = \frac{1}{12}$$

$$= 3x+6 = 60$$

$$3x = 60-6$$

$$3x = 54$$

$$x = \frac{54}{3}$$

$$\underline{x = 18}$$

$$p(y) = \frac{(2(18)+1)}{3(18)+6}$$

$$\boxed{p(y) = \frac{37}{60}} \checkmark$$

* the sum of probabilities of all possible outcomes = 1

repeated frequency = $\overset{\text{number of spins}}{\uparrow}$ n x probability

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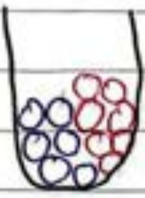
Picking 2 items:

tree diagram



Ahmed is picking 2 balls at random.

red ~~brown~~ blue $= \frac{7}{13} \times \frac{6}{13} = \frac{42}{169}$
red red
blue red
blue blue

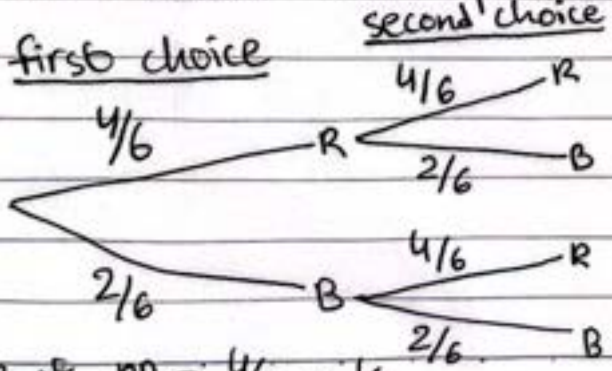
Tree diagram for independant events:

Two event A & B are independant if ~~no~~ one event happening has no effect on weather or not the other happens.

$$P(A \text{ and } B) = P(A) \times P(B)$$

*'or' means '+'

example:
a bag contains 4 red balls and 2 blue balls.
2 balls are chosen at random from the bag (with replacment). draw a tree diagram to show all possible outcomes.



outcome	probability
RR	$\frac{4}{6} \times \frac{4}{6} = \frac{16}{36}$
RB	$\frac{4}{6} \times \frac{2}{6} = \frac{8}{36}$
BR	$\frac{2}{6} \times \frac{4}{6} = \frac{8}{36}$
BB	$\frac{2}{6} \times \frac{2}{6} = \boxed{\frac{4}{36}}$

prob of BB $= \frac{4}{36} = \frac{1}{9}$

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b) probability of 2 the same color $= \frac{16}{36} + \frac{4}{36}$

$= \frac{20}{36} = \boxed{\frac{5}{9}}$

c) probability of 2 different colors =
RB or Br

$\frac{8}{36} + \frac{8}{36} = \frac{16}{36}$

$= \boxed{\frac{4}{9}}$

*total of each branch = 1

d) probability at least one red $= \frac{16}{36} + \frac{8}{36} + \frac{8}{36}$
RB BR RR

$= \frac{32}{36} = \boxed{\frac{8}{9}}$

Tree diagram for dependent events:
Two events A & B are dependent, one event happening has an effect on weather or not the others happen.

example:
a bag contains 4 red balls and 2 blue balls.
2 balls are chosen at random from the bag without replacment. draw a tree diagram to show all the possible outcomes.

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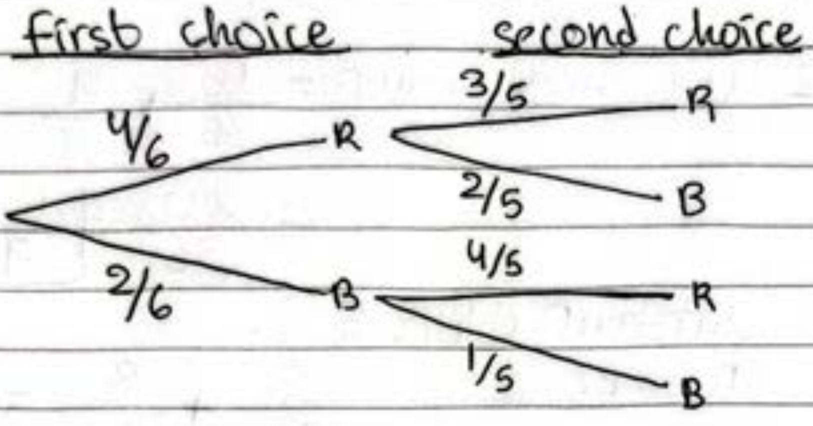


outcome	probability
RR	$\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}$
RB	$\frac{4}{6}\times\frac{2}{5}=\frac{8}{30}$
BR	$\frac{2}{6}\times\frac{4}{5}=\frac{8}{30}$
BB	$\frac{2}{6}\times\frac{1}{5}=\frac{2}{30}$

a) probability of 2 reds $=\frac{12}{30}=\boxed{\frac{2}{5}}$

Arrangments

2 balls (2 different colors)

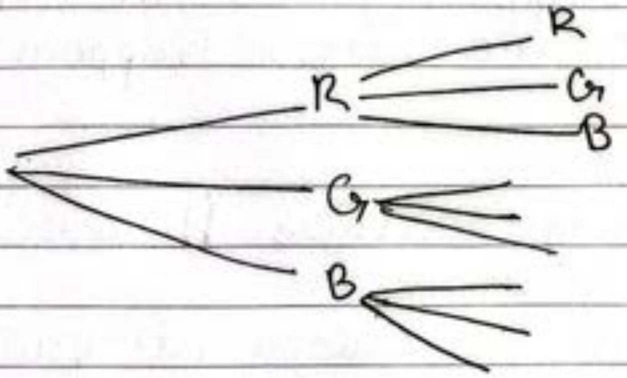
blue red

*with replacment

① BR + RB = 2(BR)

3 choices

6(RBG)
- RBG
- RGB
- GRB
- GBR
- BGR
- BRG



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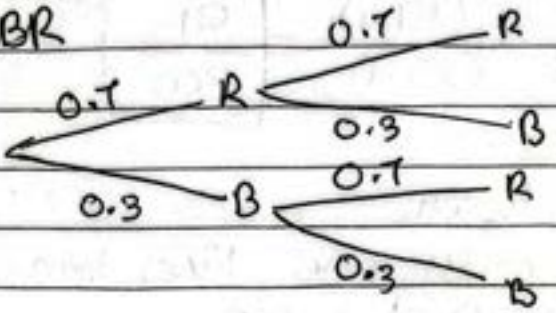
Date: 20/3/2024

Arrangments

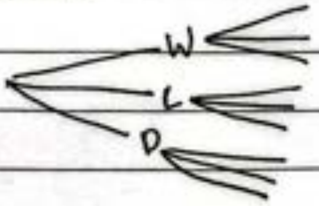
2 choices:

RB+BR
or 2BR



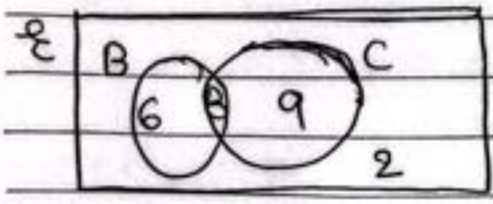
$\longrightarrow 0.7\times0.3 + 0.3\times0.7 = 0.42$
$\longrightarrow 2BR = 2(0.7\times0.3) = 0.42$

3 choices:



WLD, WDL, DWL, DLW, LWD, LDW
6WLD

Probability of venn diagram:



a) a student is chosen at random, find the probability they like chocolate and banana.

$P = \frac{8}{25}$
(denominator)

conditional

b) a student who like choc. icecream is chosen. find probability they like banana too.

$\frac{8}{(8+9)} = \boxed{\frac{8}{17}}$

c) 2 students are chosen.
find prob. they both like banana ice cream

$$\text{prob.} = \frac{14}{25} \times \frac{(14-1)}{(25-1)} = \boxed{\frac{91}{300}}$$

d) 2 students are chosen.
find prob. that <u>only one</u> likes banana.

$$BB' + B'B$$

$$\text{prob.} = \left(\frac{14}{25} \times \frac{\overset{\overset{(9+2)}{\uparrow}}{11}}{\underset{\underset{(25-1)}{\downarrow}}{24}}\right) + \left(\frac{\overset{(9+2)}{11}}{25} \times \frac{14}{24}\right) = \boxed{\frac{91}{300}}$$

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Sub: REVISION
Date: 24/3/2024

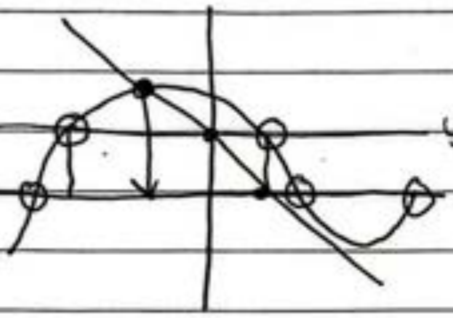
Graphs of functions

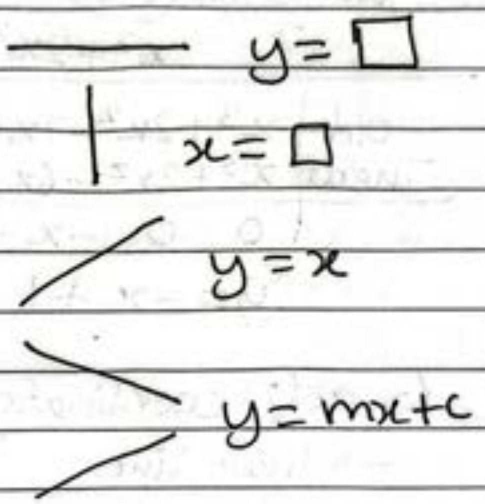
① complete the table

$y = x^3 + 2x^2 - 7x + 5$

x	-1	0	1	2
y	13	5	1	7

② plotting the points





③ solve from the graph

a) estimate the solution of

i) $x^3 + 2x^2 - 7x + 5 = 0$
ii) $f(x) = 0$
iii) $y = 0$

same thing

get the difference between both, if no change → get points that touch x-axis

$x =$ ____ , $x =$ ____ , $x =$ ____

b) estimate the solution of

i) $x^3 + 2x^2 - 7x + 5 = 3$
ii) $f(x) = 3$
iii) $y = 3$

$x =$ ____ , $x =$ ____

steps:
① compare
② draw line
③ get x-coordinates

c) by drawing a suitable straight line, estimate the solution of

$x^3 + 2x^2 - 6x + 4 = 0$

old $\cancel{x^3} + \cancel{2x^2} - 7x + 5$
− new $\cancel{x^3} + \cancel{2x^2} - 6x + 4$
$0 \quad 0 \quad -x \quad 1$

↳ $y = -x + 1$

↓ to draw

x	0	1
y	1	0

plot point, connect and get x values.

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d) by drawing a suitable straight line, solve

$$2x^3+4x^2-12x+8=0$$

↳ number before x^3 & x^2 have to be the same $\rightarrow$ $(\div 2)$

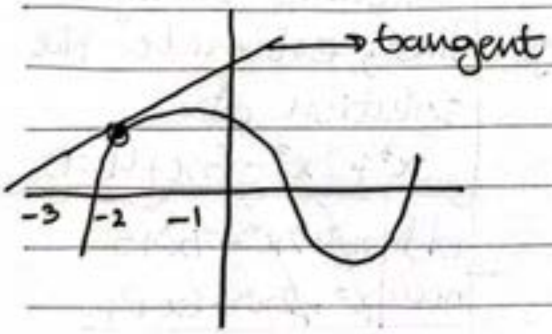
$$x^3+2x^2-6x+4=0$$

$$\begin{array}{r|l} \text{old} & x^3+2x^2-7x+5 \\ -\text{new} & x^3+2x^2-6x+4 \\ \hline & 0 \quad 0 \quad -x+1 \end{array}$$

$$y=-x+1$$

↳ get coordinates
→ draw line
→ get y's

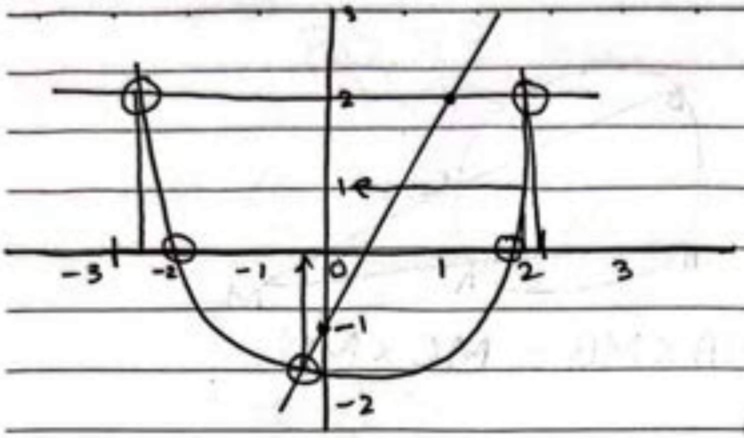
④ estimate the gradient of the curve at $x=-2$



① get 2 points on line
② get gradient using

$$m=\frac{y_2-y_1}{x_2-x_1}$$

* to make sure gradient is correct:
① shift $\rightarrow \frac{d}{dx}\square$
② write equation of curve and the given x value



find :

a) $f(2) = 1$

b) solve $= f(x) = 0$

$\hookrightarrow y=0$

$x_1=-2, x_2=2$

c) solve $f(x)=2$

$\hookrightarrow y=2$

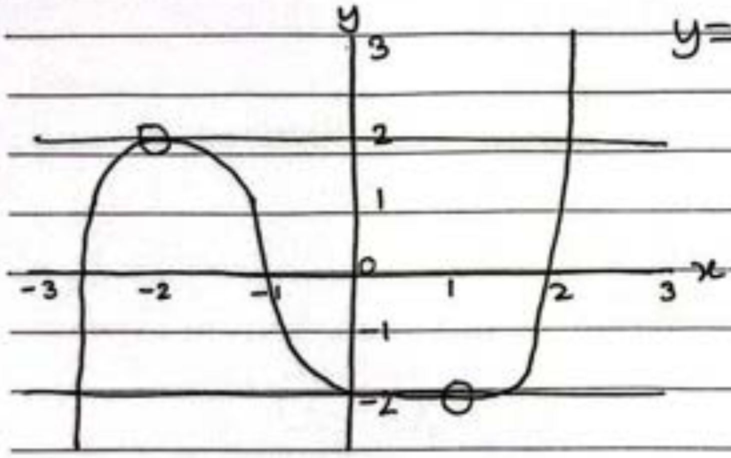
$x_1=-2.5, x_2=2.3$

d) solve $f(x)=3x-1$

$\hookrightarrow y=3x-1$

x	0	1
y	-1	2

$x=-0.3$



$y=f(x)$

$y=k$ is drawn there are 3 solutions between $f(x)$ and $y=k$ find the possible values of k.

$-2 < k < 2$

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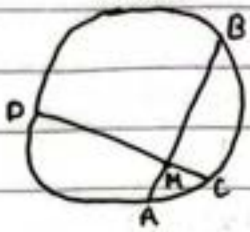
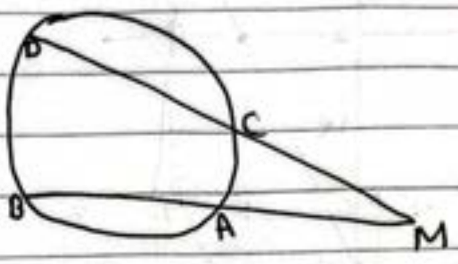
Intersecting Chords

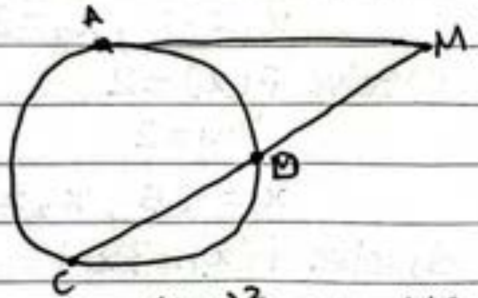
inside



$MA \times MB = MC \times MD$



$MA \times MB = MC \times MD$



$(AM)^2 = MD \times MC$

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Arithmetic Sequences

Sub:
Date: 27/3/2024

① $80, \underbrace{73}_{-7}, \underbrace{66}_{-7}, \underbrace{59}_{-7}, \_\_\_$

② $16, \underbrace{21}_{+5}, \underbrace{26}_{+5}, \underbrace{31}_{+5}, \_\_\_$

(nth term) $U_n = \underset{\text{1st term}}{\boxed{a}} + (\underset{\text{order}}{n}-1)\underset{\text{difference}}{\boxed{d}}$

*

$U_n = \underset{\cancel{\text{1st term}}\ \text{difference}}{\boxed{a}}\underset{\text{order}}{n} + \underset{\text{term zero}}{\boxed{b}}$

find nth term for:

sequence 1:-

$\rightarrow U_n = 80 + (n-1)-7$

$= 80 - 7n + 7$

$\underline{U_n = 87 - 7n}$

$\rightarrow U_n = -7n + (80+7)$

$\underline{U_n = -7n + 87}$

sequence 2:-

$\rightarrow U_n = 16 + (n-1)5$

$= 16 + 5n - 5$

$\underline{U_n = 11 + 5n}$

$\rightarrow U_n = 5n + (16-5)$

$\underline{U_n = 5n + 11}$

$$\begin{array}{cccccc} U_1 & U_2 & U_3 & U_4 & U_5 & \\ 3, & 6, & 9, & 12, & 15, & \_\_\_ \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ a & a+d & a+2d & a+3d & a+4d & \end{array}$$

* you can express any term in the sequence in terms of a & d.
by using : $U_n = a + (n-1)d$.

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Sub:

Date:

Consecutive terms.

$U_1 \quad U_2 \quad U_3$

$\underline{2k}, \underline{k+1}, \underline{3-k}$

are 3 consecutive terms

find U_{15} if $U_4 = 10$

* $\boxed{U_3 - U_2 = U_2 - U_1}$

example

here are 3 consecutive terms of an arithmetic sequence.

$\underline{3k-1}, \underline{2k+5}, \underline{5k+3}$

$5 \xrightarrow[d=4]{} 9 \xrightarrow[d=4]{} 13$

if $U_7 = 10$, find U_{20}.

$\rightarrow 5k+3-(2k+5) = 2k+5-(3k-1)$

$3k-2 = -k+6$

$3k+k = 6+2$

$4k = 8$

$\underline{k = 2}$

$\underline{d = 4}$

$U_7 = a+(7-1)4$

$10 = a + 6 \times 4$

$10 = a + 24$

$\underline{-14 = a}$

$U_{20} = -14 + (20-1)4$

$= -14 + 19 \times 4$

$= -14 + 76$

$\underline{U_{20} = 62}$

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Sub:
Date:

Arithmetic Series → sum

sequence : 2, 4, 6, 8, 10, ____

series : 2 + 4 + 6 + 8 + 10 + ____

$$\underset{\text{sum}}{S_n} = \underset{\text{number of turns}}{\frac{\boxed{n}}{2}}\left[2\underset{\text{1st term}}{a}+(n-1)\underset{\text{difference.}}{d}\right]$$

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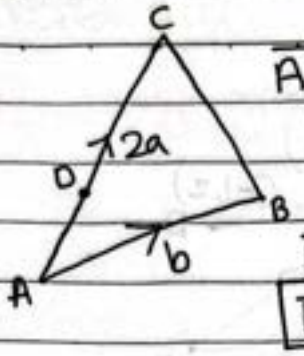
Sub:

Date: 31/3/2024

Vectors



$\overrightarrow{AC} = 2a$

$AD : DC = 1 : 3$

find $\overrightarrow{BC}$ and $\overrightarrow{BD}$?

$\overrightarrow{BC} = -b + 2a$

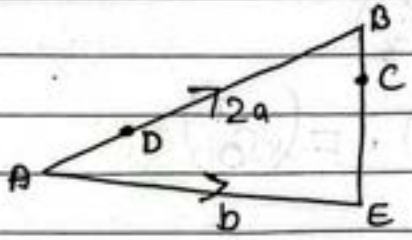
$\boxed{\overrightarrow{BC} = 2a - b}$

$\overrightarrow{BD} = -b + \frac{1}{4}(2a)$

$\overrightarrow{BD} = -b + \frac{1}{2}a$

$\boxed{\overrightarrow{BD} = a - b}$

$\boxed{\frac{1}{2}}$



$\overrightarrow{AB} = 2a$

$\overrightarrow{AE} = b$

$\overrightarrow{AD} : \overrightarrow{DB} = 1 : 3$

$\overrightarrow{EC} : \overrightarrow{CB} = 2 : 1$

find $\overrightarrow{DC}$?

$\overrightarrow{DB} = \frac{3}{4}(2a)$

$= \underline{\frac{3}{2}a}$

$\overrightarrow{BC} = -\frac{1}{3}(-b + 2a)$

$= \underline{\frac{1}{3}b - \frac{2}{3}a}$

$\overrightarrow{DC} = \frac{3}{2}a + \frac{1}{3}b - \frac{2}{3}a$

$= \underline{\frac{1}{3}b + \frac{5}{6}a}$

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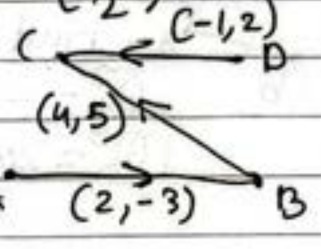
Sub:
Date: 3/4/2024

Continue Vectors

column vectors

ex $\overrightarrow{AB}=\begin{pmatrix}2\\-3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}4\\5\end{pmatrix}$, $\overrightarrow{DC}=\begin{pmatrix}-1\\2\end{pmatrix}$

find



a) $\overrightarrow{AC}$

$$\begin{pmatrix}2\\-3\end{pmatrix}+\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}$$

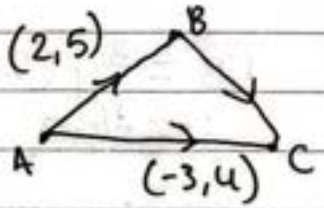
b) $\overrightarrow{DA}$

$$\begin{pmatrix}-1\\2\end{pmatrix}+-\begin{pmatrix}4\\5\end{pmatrix}+-\begin{pmatrix}2\\-3\end{pmatrix}$$

$$\begin{pmatrix}-1\\2\end{pmatrix}+\begin{pmatrix}-4\\-5\end{pmatrix}+\begin{pmatrix}-2\\+3\end{pmatrix}=\begin{pmatrix}-7\\0\end{pmatrix}$$

ex $\overrightarrow{AB}=\begin{pmatrix}2\\5\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}-3\\4\end{pmatrix}$

find magnitude of $\overrightarrow{BC}$



$$\overrightarrow{BC}=\begin{pmatrix}-2\\-5\end{pmatrix}+\begin{pmatrix}-3\\4\end{pmatrix}=\begin{pmatrix}-5\\-1\end{pmatrix}$$

$$|\overrightarrow{BC}|=\sqrt{-5^2+-1^2}$$
$$=\sqrt{26}$$

point and vector

$\overrightarrow{AB}$ = ending point - starting point

Sub:

Date:

Differentiation

~~finding gradient of tangent of curve~~

$\frac{dy}{dx}$

↳ steps:

- multiply power by coefficient.
- subtract one from the power.

find $\frac{dy}{dx}$?

① $y = 3x^2 + 2x - 7$

↳ $\underline{6x + 2}$

② $y = 6x^3 + 3x(x-4) \rightarrow y = 6x^3 + 3x^2 + 12x$

↳ $\underline{18x^2 + 6x \cancel{+} 12}$

③ $y = 3x^2 + \frac{4}{5x} \longrightarrow y = 3x^2 + \frac{4}{5}x^{-1}$

↳ ~~$3x^2 + 4 \times 5x^{-1}$~~

✗ ↳ ~~$6x - 5x^{-2}$~~

↳ ~~$6x$~~

↳ $6x^{\cancel{3}} - \frac{4}{5}x^{-2}$

↳ $\underline{6x^{\cancel{3}} - \frac{4}{5x^2}}$ ☆

find gradient of tangent

① get $\frac{dy}{dx}$

② replace x-values in $\frac{dy}{dx}$

ex

$y = x^3 - 4.5x^2 + 5$
has a positive gradient, find range of positive values.

$\frac{dy}{dx} = 3x^2 - 9x$

$0 < 3x^2 - 9x$

$0 \leq 3x(x-3)$

$3x = 0$	$x - 3 = 0$
$x = 0$	$x = 3$

$x > 3$ or $x < 0$

stationary points

① find $\frac{dy}{dx}$

② put $\frac{dy}{dx} = 0$

③ solve for x

④ replace x in $f(x)$ to get y-coordinates.

ex $y = 2x^3 - 6x^2 + 4$ has 2 turning points, find their coordinates.

$\frac{dy}{dx} = 6x^2 - \overset{12}{6}x$

$0 = 6x^2 - 12x$

$0 = 6x(x-2)$

$6x = 0$	$x - 2 = 0$
$x = 0$	$x = 2$

at $x = 0$	at $x = 2$
$y = 4$	$y = -4$

$(0, 4)$ $(2, -4)$

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Sub:
Date: 6/4/2024

Differentiation

*max or min value of volume *revise coordinate
area geometry &
variable. graphs.

1)* find an express for volume/area/variable.

2) find $\frac{dv}{dx}$ / $\frac{dA}{dx}$ / $\frac{dy}{dx}$

3) put $\frac{dv}{dx}$ / $\frac{dA}{dx}$ / $\frac{dy}{dx}$ $= 0$

4) solve for x

5) replace x in volume/area/variable expression.

* to verify / explain that volume/area/volume is max or min.

1) find $\frac{d^2v}{dx^2}$ (2^{nd} derivative)

2) replace x value.

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Sub:
Date:

Kinematics

displacment → velocity → acceleration

s $\quad$ v $\quad$ a

x $\quad$ $v = \frac{ds}{dt}$ $\quad$ $a = \frac{dv}{dt}$

when does $V=0$?

a) particle is stationary
b) particle is at rest
c) particle stopped at particle A
d) particle is instantaneosly at rest
e) particle changed /reversed direction

*find the displacment :

1) find $v = \frac{ds}{dt}$

2) $0 = V$

3) solve for t

4) replace t in s.

* find max or minimum speed:

1) Find $v = \frac{ds}{dt}$

2) find $a = \frac{dv}{dt}$

3) $a=0$

4) solve for t

5) replace t in v

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Sub:
Date:

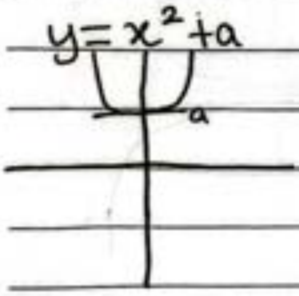
Identifying graphs

Square

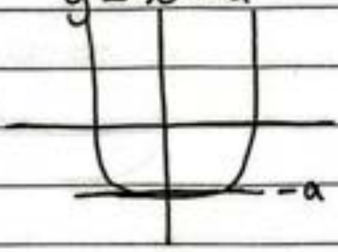
$y = x^2$

$y = -x^2$

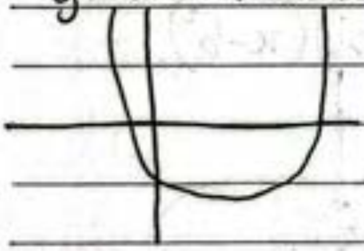
$y = x^2 + a$



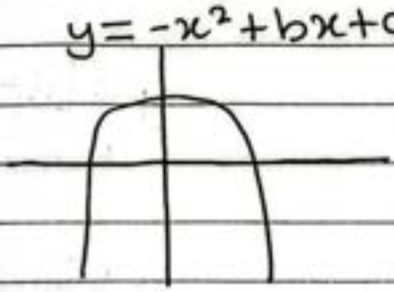
$y = x^2 - a$



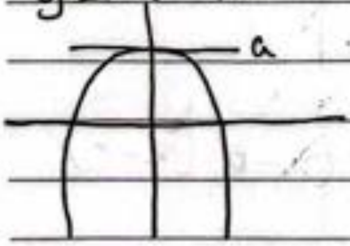
$y = ax^2 + bx + c$



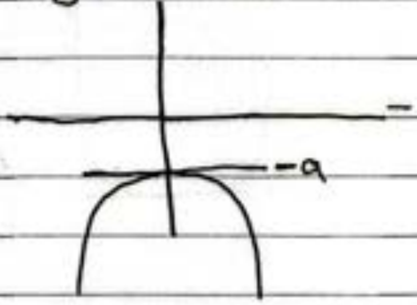
$y = -x^2 + bx + c$



$y = a - x^2$



$y = -a - x^2$



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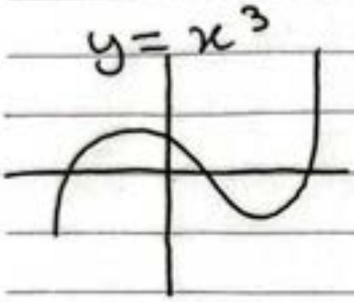
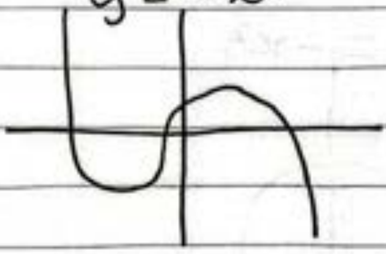
cubic

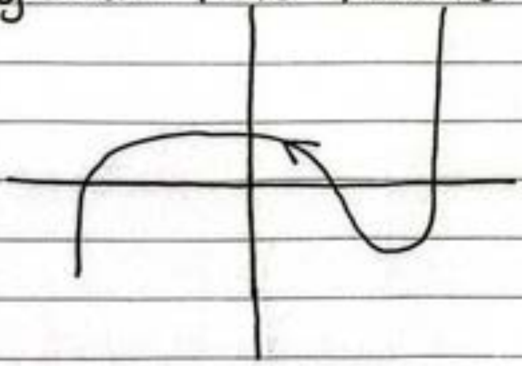
$y = x^3$



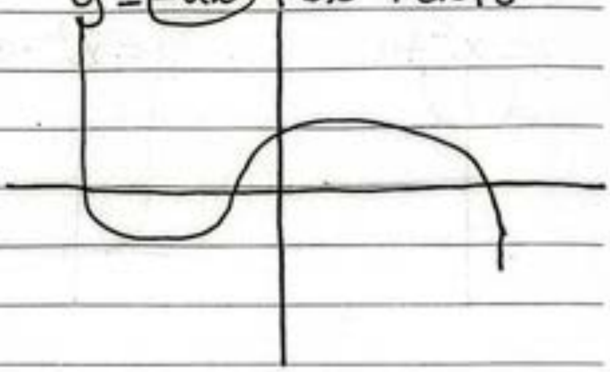
$y = -x^3$



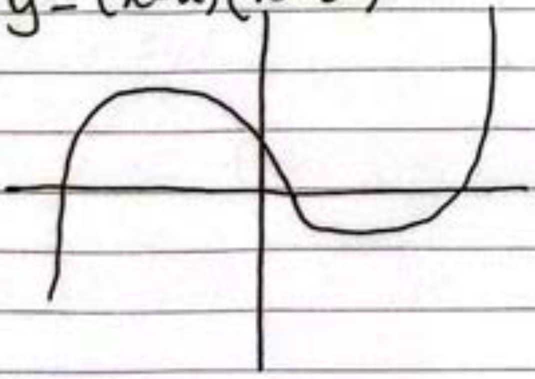
$y = ax^3 + bx^2 + cx + d$



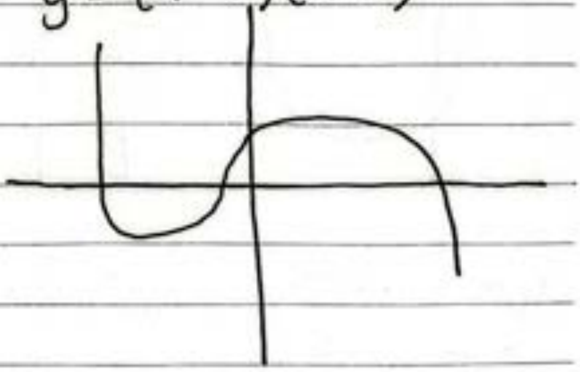
$y = \boxed{-ax^3} + bx^2 + cx + d$



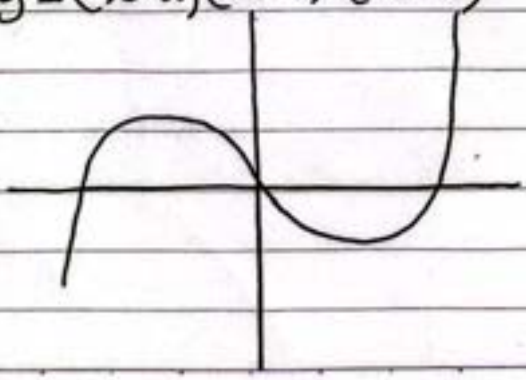
$y = (x-a)(x-b^2)$



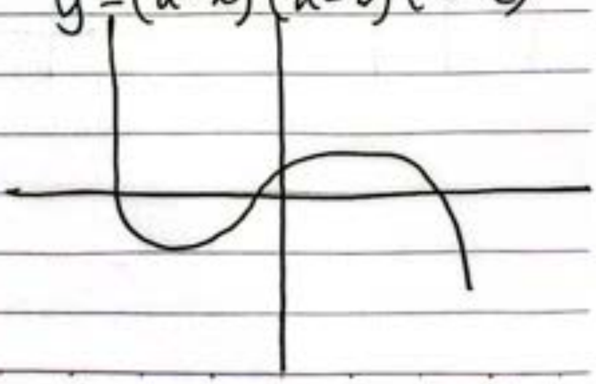
$y = (a-x)(x-b^2)$



$y = (x-a)(x-b)(x-c)$



$y = (a-x)(x-b)(x-c)$



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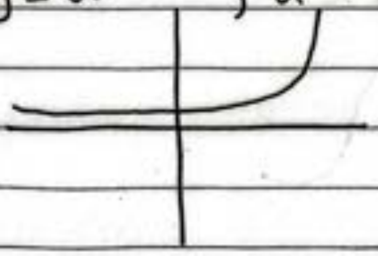
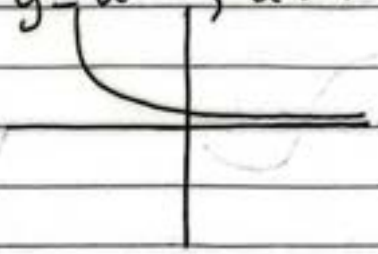
exponential

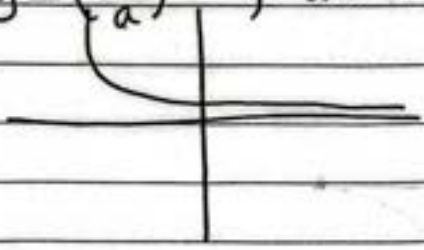
$y=a^{x}$, $a>1$



$y=a^{-x}$, $a>1$

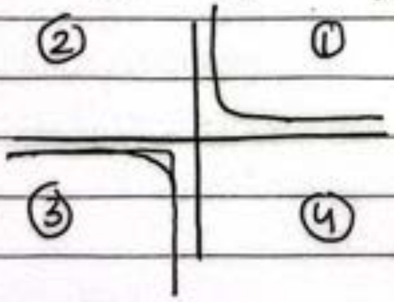


$y=\left(\frac{1}{a}\right)^{x}$, $a>1$

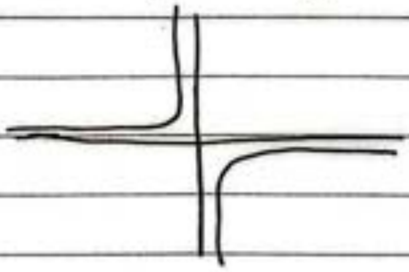


reciprocal

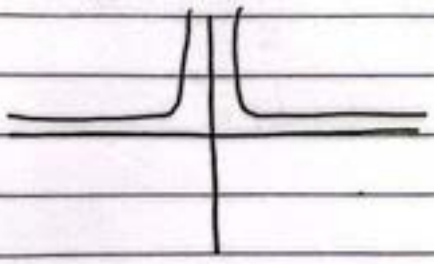
$y=\frac{1}{x}, \frac{a}{x}, \frac{\square}{x}$



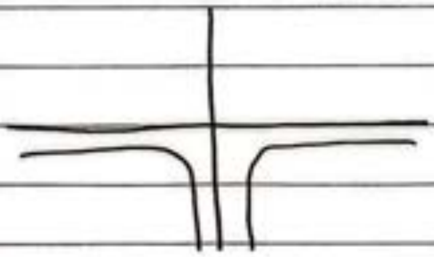
$y=\frac{-1}{x}, \frac{-a}{x}, \frac{-\square}{x}$



$y=\frac{1}{x^2}, \frac{a}{x^2}, \frac{\square}{x^2}$



$y=\frac{-1}{x^2}, \frac{-a}{x^2}, \frac{-\square}{x^2}$



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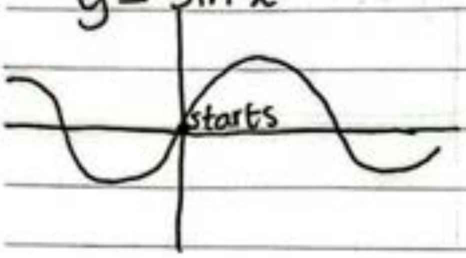
Sub:

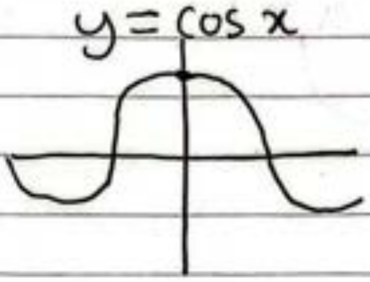
Date:

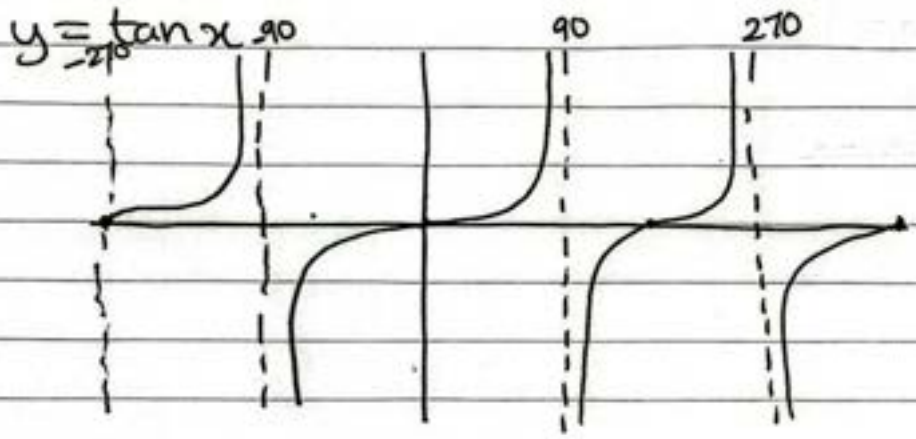
trignometric graphs

$y = \sin x$



$y = \cos x$





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Sub:
Date: [redacted] 20/4/2024

[redacted]
[redacted]

[redacted]
Statistics :

mon 1H 4pm
thursday 2H 4pm

discrete (whole numbers)

scores : 20, 17, 11, 12, 19, 11

mean = [redacted] $\frac{\text{sum}}{\text{no. of values}}$
mode = most repeated
median = middle value (after order)
range = highest - lowest

frequency table

scores	10	11	12	13	14	15
frequency	3	7	8	2	10	5

n → no. of data values

frequency table ans.

$$\text{mean} = \frac{\leq xf}{\leq f} = \frac{10\times3 + 11\times7 + 12\times8 + 13\times2 + 14\times10 + 15\times5}{3+7+8+2+10+5}$$

$$= \frac{444}{35}$$

modal = 14

$$\text{median} = \frac{n}{2} = \frac{\leq f}{2} = \frac{35}{2} = 17.5^{th} \simeq 18th$$

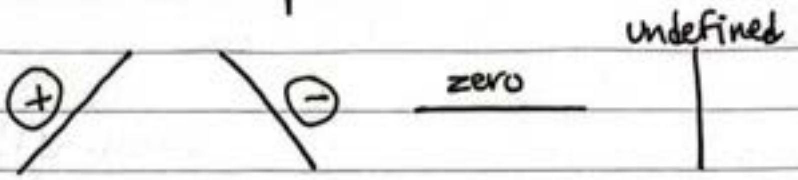
$$= 12$$

Sub:

Date: 30/4/2024

Coordinate Geometry

gradient : steepness



→ $(x_1,y_1)(x_2 y_2)$

$$\underset{\swarrow \text{ gradient}}{\boxed{m}} = \frac{y_2-y_1}{x_2-x_1}$$

line → $y = mx + c$

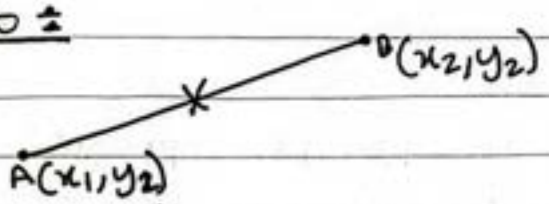
→ $ax+by+c=0$

$by = -ax - c$

$$y = \underset{\downarrow}{\boxed{-a/b}}\,x - c/b$$

gradient.

midpoint :



$$M = \left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$$

distance between 2 points (length of).

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

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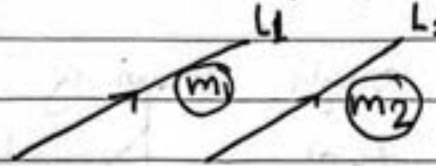
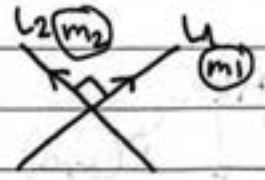
parallel and perpendicular lines :



$m_1 = m_2$

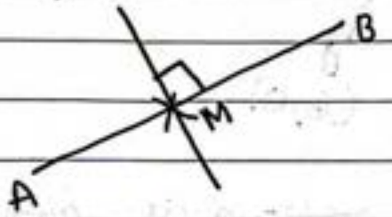
parallel lines



$m_1 \times m_2 = -1$

perpendicular lines
(flip and switch sign)

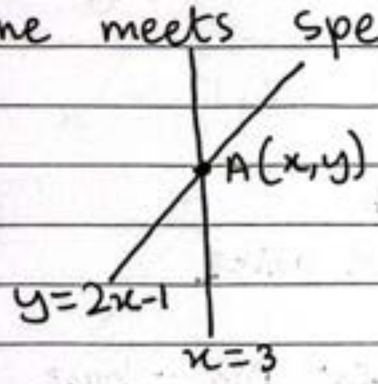
perpendicular bisector :



→ perpendicular to $\overrightarrow{AB}$.

→ passes through the midpoint.

line meets special line : vertical $(x=a)$ horizontal $(y=a)$



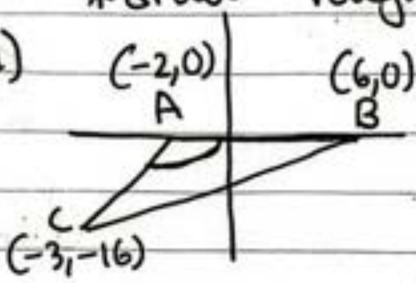
$x = 3 \; ; \; y = 2(3) - 1 = 5$

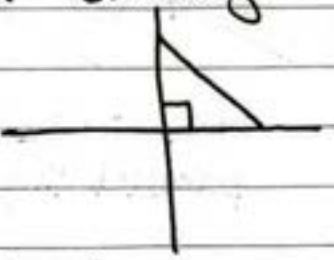
$A(3,5)$

area of triangles:

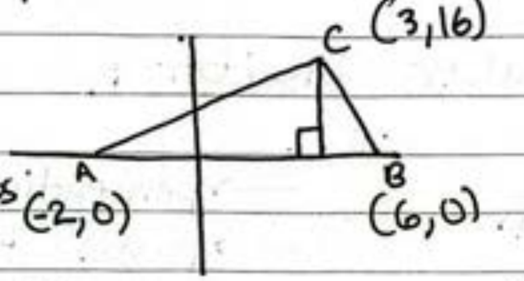
ΔABC → (x,y), (a,b), (c,d)

*draw rough drawing



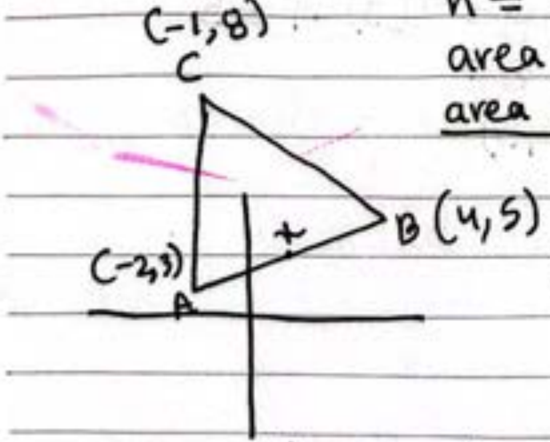


$b = 6 - -2 = 8$
$h = 16$
$\text{area} = 1/2 \times 8 \times 16$
area = 64 square units



* if on same ~~axe~~ axis and you want length subtract big from small.

$b = 6 - -2 = 8$
$h = 16$
$\text{area} = 1/2 \times 8 \times 16$
area = 64 square units



① * consider AB = base
$AB = \sqrt{(4--2)^2 + (5-3)^2}$
$AB = 2\sqrt{10}$
$x =$ [illegible]

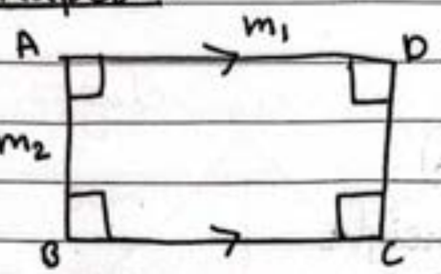
- x is midpoint of AB

$\frac{-2+4}{2}, \frac{3+5}{2}$

② $x = (1, 4)$
$Cx = \sqrt{(1--1)^2 + (4-8)^2}$
$Cx = 2\sqrt{5}$

③
$\text{area} = 2\sqrt{10} \times 2\sqrt{5} \times 1/2$
$\text{area} = 10\sqrt{2}$

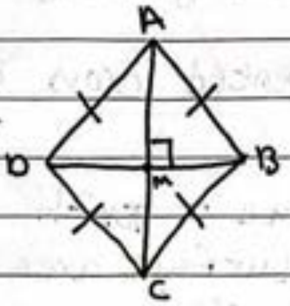
shapes :



rectangle :-

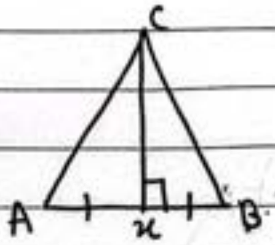
$(AD)\ m_1 = (BC)\ m_1$

$(AD)\ m_1 \times (BC)\ m_1 = -1$



rhombus :-

diagonals are perpendicular.

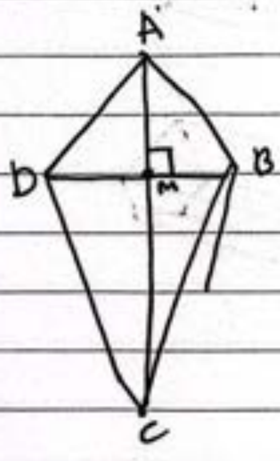


isoscles triangle :-

CX : line of symetry

$\Downarrow$

perpenndicular bisector.



kite :-

AB = AD

CB = CD

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Mensuration

Sub:
Date: 30/4/2024

cube : (all faces are squares)

volume = length^3
surface area = $6(\text{length})^2$

cuboid : (rectangular prism)

volume = length x width x height
surface area = 2[length x width + length x height + width x height]

prism : 3d shape with repeated cross sectional face.

- circular :
  cylinder

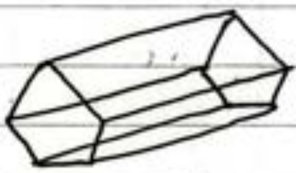
volume = $\pi r^2 h$
curved surface area = $2\pi rh$
total surface area = $2\pi rh + 2\pi r^2$

- dimensional prism :
  (cross section is polygon)

triangular prism

cuboid

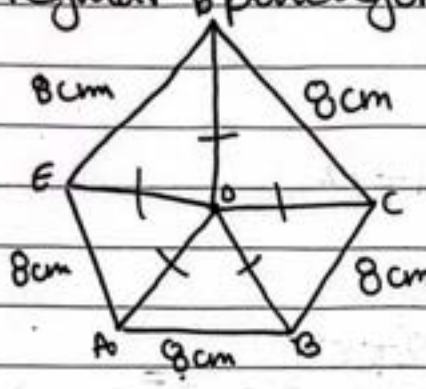
pentagonal prism

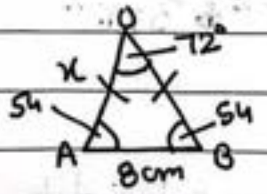


hexagonal pirism

area of polygons:

regular pentagon





$\frac{360}{5} = 72^{\circ}$

$\frac{(180-72)}{2} = 54$

$\frac{8}{\sin 72} = \frac{x}{\sin 54}$

$\frac{8 \times \sin 54}{\sin 72} = x$

$6.81 = x$

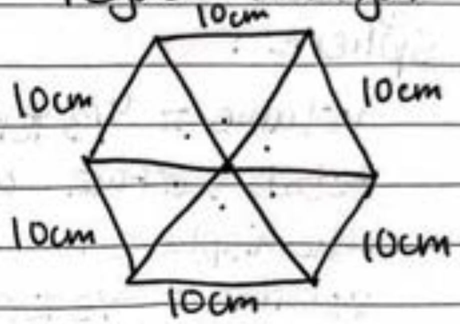
$area = \frac{1}{2} \times 8 \times 6.81 \times \sin 54$

$area = 22.04$

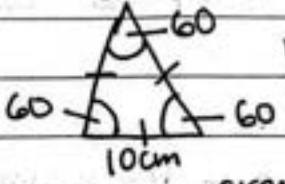
$total = 5 \times 22.04$

$total = 110$

regular hexagon



$\frac{360}{6} = 60^{\circ}$



$\frac{180-60}{2} = 60$

$area = \frac{1}{2} \times 10 \times 10 \times \sin 60$

$area = 25\sqrt{3}$

$total = 6 \times 25\sqrt{3}$

$total = 260$

Sub:
Date:

volume of prism = area of cross section x length

sphere :

volume = $4/3 \pi r^3$

total surface area = $4\pi r^2$

- hemisphere :

volume = $2/3 \pi r^3$

curved surface area = $2\pi r^2$

total surface area = $\pi r^2 + 2\pi r^2 = 3\pi r^2$

pyramid :

volume = 1/3 x area of the base x height

total surface area (square) = area of base + 4 x Δ

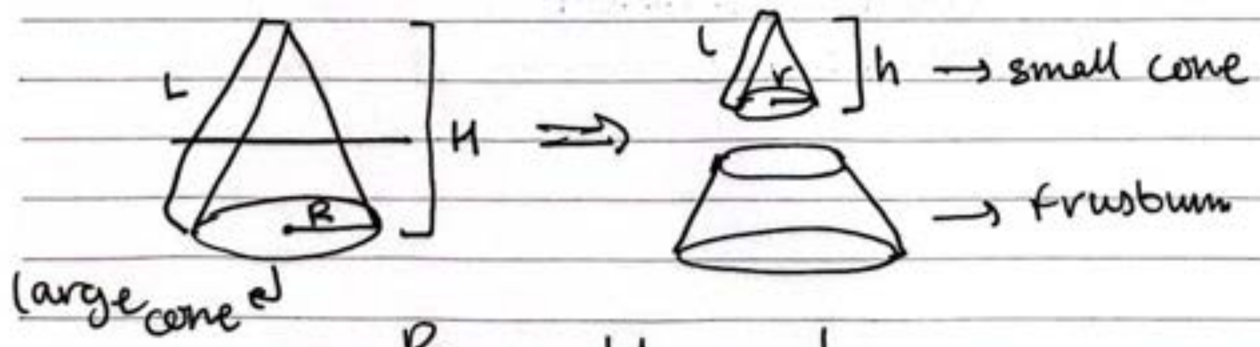
total surface area (rectangle) = area of base + $2\Delta_1 + 2\Delta_2$

cone :

volume = $1/3 \pi r^2 h$ $L = \sqrt{h^2 + r^2}$

curved surface area = $\pi r \boxed{L}$ → slanted height

total surface area = $\pi r^2 + \pi r l$

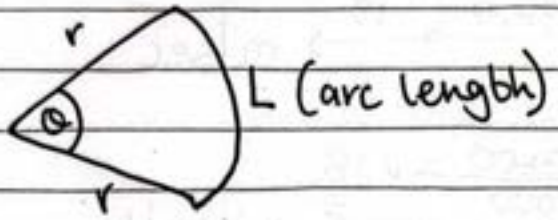


$$\frac{R}{r} = \frac{H}{h} = \frac{L}{l}$$

volume of frustum = volume large - volume small

(c.s.a) frustum = (c.s.a) large - (c.s.a) small

sectors:

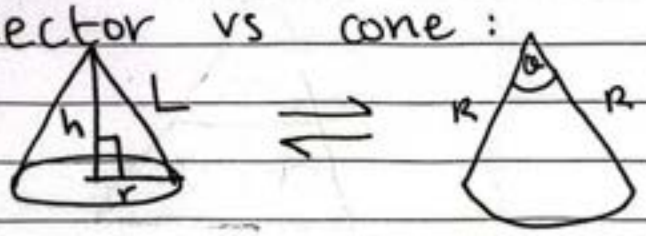


arc length ← $L = \frac{\theta}{360} \times 2\pi r$ or $\frac{\theta}{360} = \frac{L}{2\pi r}$

area of sector ← $A = \frac{\theta}{360} \times \pi r^2$ or $\frac{\theta}{360} = \frac{A}{\pi r^2}$

perimeter of sector ← $P = 2r + L$

sector vs cone :



slant height (cone) = radius of sector
curved surface area (cone) = area of sector.
circumference of the base = arc length

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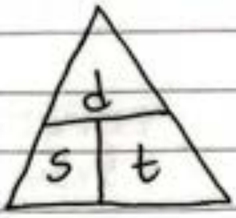
Sub:

Date: 2/5/2024

Units of conversion

speed, distance, time



$\text{km/hr} \xrightarrow{\times\frac{1000}{3600} = \times\frac{5}{18}} \text{m/sec}$

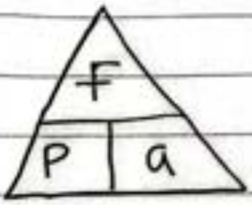
$d = s \times t$

$t = \frac{d}{s}$

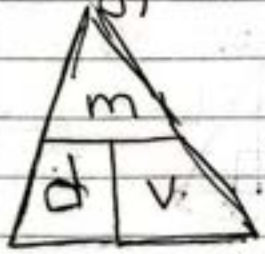
$s = \frac{d}{t}$

$\text{m/sec} \xrightarrow{\times\frac{3600}{1000} = \times\frac{18}{5}} \text{km/h}$

pressure, force, area



density, mass, volume



Sketching graphs :

to sketch any function :-

① x - intercepts $(y=0)$

② y - intercepts $(x=0)$

③ turning point (completing the square).

Drawing and bisecting angles:

Perpendicular bisetor.

steps:- (angle).

① place compass in ~~circle~~ center
② open compass to a little more than half
③ draw 2 arcs on each side
④ on the new arcs, place the compass and draw 2 more arcs in the middle with same size of step 3.
⑤ draw a line from center to meet intersection of arcs.

steps:- (line)

① place compass on one end of the line.
② open compass more than half.
③ draw an arc at the center from both sides.
④ draw a line to connect each arc.

steps:- (triangle).

① draw the first line as base with given length.
② draw given angle.
③ draw given side
④ connect the triangle.

all sides :-

① draw given side
② open compass to second side's length
③ draw arc.
④ repeat for other missing.
⑤ point of intersection of arcs is third missing point.
⑥ connect all sides as normal.

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Coordinate Geometry Important

If you have a curve and want coordinates:

① we make both equations y subject.

② equate equations together.

OR

① make one variable as subject

② substitute into other.

to get line of symetry from completing the square:

① whatever is inside the bracket reverse its sign and is the line.

kite has perpendicular diagonals.

to get intersection between $y=mx+c$ with $x=a$ or $y=b$:

substitute the value you have into equation.

area of triangle:

① get both gradients.

② get both line equations.

③ substitute given values in ~~secon~~ targetted lines. to obtain coordinates. (3rd coordinate is given or origin)

④ sketch the triangle for proper understanding.

⑤ consider the line with 2 points as base (same axis).

⑥ subtract axis value from coordinates on the base giving base value.

⑦ Y coordinate should be height if starting is origin.

⑧ area should be obtained using area = ½bh and should be in square units.

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We need exact coordinates:

① sketch for understanding.
② make sure given line is in the form $y=mx+c$.
③ obtain gradients from given line.
④ get equation of perpendicular line.
⑤ if question is asking intersect x axis → $y=0$
intersect y axis → $x=0$
⑥ substitute value of step 5 into perpendicular line ~~eqt~~ equation

missing coordinates:

① sketch for understanding.
② get gradient or use given to get perpendicular gradient.
③ place missing coordinates equalling p. gradient.
④ we need second equation using data we have.
⑤ substitute ~~at~~ step 3 after making one variable as a subject.
⑥ solve algebraically.
⑦ check if any restriction on answer.

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